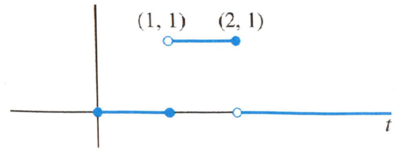
10.1 Exercises - Solutions

Problem 1 Apply the definition $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$ to directly find the Laplace transform of

the following graph: $f(t) =$



Solution: $\mathcal{L}\{f(t)\} = \int_0^{\infty}[e^{-st}f(t)]dt = \int_1^2[e^{-st} \cdot 1]dt = \left[-\frac{e^{-st}}{s}\right]_1^2 = \frac{e^{-s}-e^{-2s}}{s}$.

Problem 2 Use the common transforms to find the transform of $f(t) = t^{\frac{3}{2}} - e^{-10t}$.

Solution: Because of linearity: $\mathcal{L}\left\{t^{\frac{3}{2}} + e^{-10t}\right\} = \mathcal{L}\left\{t^{\frac{3}{2}}\right\} + \mathcal{L}\{e^{-10t}\}$.

Using the table:

$t^a \ (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$

$\mathcal{L}\left\{t^{\frac{3}{2}}\right\} + \mathcal{L}\{e^{-10t}\} = \frac{\Gamma\left(\frac{5}{2}\right)}{s^{\frac{5}{2}}} + \frac{1}{s+10}$.

Recall: "$\Gamma(x+2) = (x+1)\Gamma(x+1) = x(x+1)\Gamma(x)$"

So, $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \left(\frac{3}{2} \cdot \frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{\pi}$.

And, $\mathcal{L}\left\{t^{\frac{3}{2}} + e^{-10t}\right\} = \frac{3\sqrt{\pi}}{4s^{\frac{5}{2}}} + \frac{1}{s+10}$.

Problem 3 What is the inverse Laplace transform of the function $\frac{9+s}{4+s^2}$?

Solution: $\mathcal{L}^{-1}\left\{\frac{9+s}{4+s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{9}{4+s^2} + \frac{s}{4+s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{9}{4+s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{4+s^2}\right\}$.

Using the table:

$\cos kt$	$\frac{s}{s^2+k^2}$	$s > 0$
$\sin kt$	$\frac{k}{s^2+k^2}$	$s > 0$

$\mathcal{L}^{-1}\left\{\frac{9}{4+s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{4+s^2}\right\} = \frac{9}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} = \frac{9}{2}\sin 2t + \cos 2t$.

[Adapted from Differential Equations & Linear Algebra, Edwards & Penny]