9.2 Exercises

Problem 1 The system: $\frac{dx}{dt} = x - 2y + 1, \quad \frac{dy}{dt} = x + 3y - 9,$ has a single critical point (x_0, y_0). Apply Theorem 2 (Stability of Almost Linear Systems, page 538) to classify this critical pt as to type and stability. Verify your conclusion by constructing a phase portrait for the given system.

Problem 2 Investigate the type of the critical point $(0,0)$ of the almost linear system:
$\frac{dx}{dt} = 5x - 3y + y(x^2 + y^2), \quad \frac{dy}{dt} = 5x + y(x^2 + y^2).$

Verify your conclusion by constructing a phase portrait.

Problem 3 The term bifurcation generally refers to something "splitting apart." With regard to DEQss or systems involving a parameter, it refers to abrupt changes in the character of the sols as the parameter is changed continuously. The problem below illustrates a sensitive case in which small perturbations (changes) in the coefficients of this almost linear system can change the type or stability (or both) of a critical pt.

$$\frac{dx}{dt} = -x + \varepsilon y, \qquad \frac{dy}{dt} = x - y.$$

Show that the critical pt $(0,0)$ is: *a*) a stable spiral pt if $\varepsilon < 0$; *b*) a stable node if $0 \le \varepsilon < 1$.

Problem 4 **What types of eigenvalues might be associated with these phase portraits of NONLINEAR systems**?

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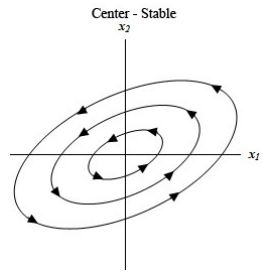
<td> 
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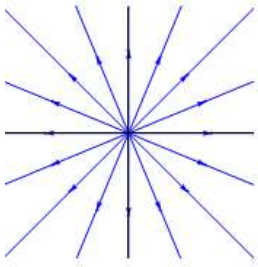
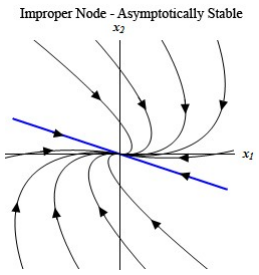
<td>  </td>
<td> 
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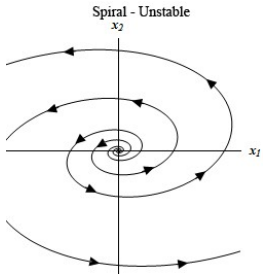
<td> 
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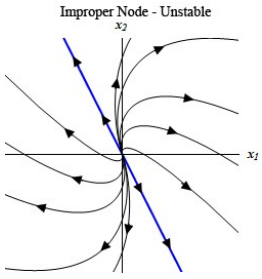
<td> 
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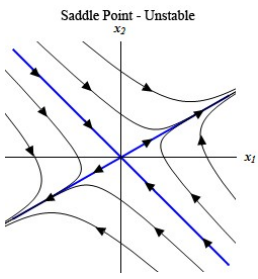
<td> 
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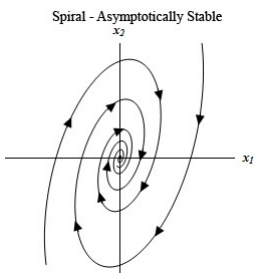
<td> 
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