Name________________________________

9.1 Exercises

Problem 1 Find the critical point or points of $\frac{dx}{dt} = 1 - y^2, \qquad \frac{dy}{dt} = x + 2y.$

Problem 2 Find the equilibrium solution $x(t) \equiv x_0$ of: $x'' + (x^2 - 1)x' + x = 0$. Construct a phase portrait and direction field (Geogebra.org/m/utcMvuUy) for the equivalent 1st-order system: $x' = y, \quad y' = -(x^2 - 1)y - x$. Ascertain whether the critical point $(x_0, 0)$ looks like a center, saddle point, or spiral point.

Problem 3 Solve the following system to determine whether the critical point $(0, 0)$ is stable, asymptotically stable, or unstable. Construct a phase portrait and direction field for the given system (Geogebra.org/m/utcMvuUy). Ascertain the stability or instability of each critical point, and identify it visually as a node, a saddle point, a center, or a spiral point. $\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -5x - 4y$

Problem 4 Given the system: $\frac{dx}{dt} = y^3 e^{x+y}, \quad \frac{dy}{dt} = -x^3 e^{x+y}.$

Solve the equation: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-x^3 e^{x+y}}{y^3 e^{x+y}}$ to find the trajectories of the given system.

Construct a phase portrait and direction field for the system (by hand, or Geogebra.org/m/utcMvuUy).

Identify visually the apparent character and stability of the critical point $(0, 0)$ of the system.