7.4 Exercises

Problem 1 For the system: $x_1' = 4x_1 + x_2,\ x_2' = 6x_1 - x_2$, categorize the eigenvalues ($\lambda_{1,2} = -2, 5$) and eigenvectors ($\vec{v}_1 = [1 \quad -6]^T$ and $\vec{v}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$) of the coefficient matrix **A** according to Fig. 7.4.16 in the textbook, and sketch the phase portrait of the system by hand. Then use a computer system or graphing calculator (WolframAlpha.com: StreamPlot[{4x1 + x2, 6x1 - x2}, {x1, -2, 2}, {x2, -2, 2}]) to check your answer.

Problem 2 For system: $x_1' = x_1 - 5x_2,\ x_2' = x_1 - x_2$, categorize e-vals ($\lambda_{1,2} = \pm 2i$) and e-vecs ($\vec{v}_{1,2} = [1 \pm 2i \quad 1]^T$) of **A** according to Fig. 7.4.16 in the textbook, and sketch the phase portrait of the system by hand. Then use a computer system or graphing calculator (WolframAlpha.com: StreamPlot[{x1 -5x2, x1 - x2}, {x1, -2, 2}, {x2, -2, 2}]) to check your answer.

Problem 3 For the system: $x_1' = x_1 - 5x_2, \quad x_2' = x_1 + 3x_2$, categorize e-vals ($\lambda_{1,2} = 2 \pm 2i$) and e-vecs ($\vec{v}_{1,2} = [1 \pm 2i \quad -1]^T$) of **A** according to Fig. 7.4.16 in the textbook, and sketch the phase portrait of the system by hand. Then use a computer system or graphing calculator (WolframAlpha.com: StreamPlot[{x1 -5x2, x1 + 3x2}, {x1, -2, 2}, {x2, -2, 2}]) to check your answer.

Problem 4 The following phase portrait corresponds to a linear system of the form $\vec{x}' = \mathbf{A}\vec{x}$ in which the matrix **A** has two linearly independent eigenvectors. Determine the nature of the eigenvalues and eigenvectors. (For example, is it that the system has pure imaginary eigenvalues? Or does it has real eigenvalues of opposite sign; or an eigenvector associated with the positive eigenvalue is roughly $\begin{bmatrix} 2 & -1 \end{bmatrix}^T$, etc.)