

6.3 Exercises

Problem 1 A city/suburban population transition matrix \mathbf{A} is given.

Find the resulting long-term distribution of a constant total population between the city and its suburbs.

$$\begin{aligned} C_{k+1} &= 0.8C_k + 0.1S_k \\ S_{k+1} &= 0.2C_k + 0.9S_k \end{aligned} \quad \mathbf{A} = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$$

Problem 2 This problem deals with a fox-rabbit population.

Initially, there are F_0 foxes and R_0 rabbits; after k months, there are F_k foxes and R_k rabbits.

It involves the transition matrix $\mathbf{A} = \begin{bmatrix} 0.6 & 0.5 \\ -r & 1.2 \end{bmatrix}$ where the kill rate r is 0.175.

Show that in the long term, the populations of foxes and rabbits both die out.

Problem 3 Suppose that $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Show that $\mathbf{A}^{4n} = \mathbf{I}$, $\mathbf{A}^{4n+2} = -\mathbf{I}$, and $\mathbf{A}^{4n+3} = -\mathbf{A}$, for every positive integer n .