

5.2 Exercises

Problem 1. Functions $y_1 = x$ and $y_2 = x^2$ are linearly independent solutions (on the entire real line) of the equation $x^2y'' - 2xy' + 2y = 0$. Verify that $W(x, x^2) = 0$ when $x = 0$. Why do these observations not contradict second part of the Wronskian of Solutions Theorem?

Problem 2. Use the Wronskian to show that the functions $y_1(x) = e^x$, $y_2(x) = x^{-2}$, $y_3(x) = x^{-2} \ln x$ are linearly independent on $(0, \infty)$.

Problem 3. Use the Wronskian to prove that the functions $\{x, \cos(\ln x), \sin(\ln x)\}$ are linearly independent on the interval $x > 0$.