5.1 Exercises - Solutions 💪

Problem 1 Find the general solution: $2y'' + 3y' = 0.$

$2r^2 + 3r = 0.$

$r(2r+3) = 0.$

$r = 0, \ -\frac{3}{2};$

$y(x) = c_1 + c_2 e^{-\frac{3}{2}x}.$

Problem 2 Given the general solution $y(x) = c_1 e^{10x} + c_2 e^{-10x}$ of a homogeneous second order DEQ, find the DEQ in the form $ay'' + by' + cy = 0$ with constant coefficients.

$(r-10)(r+10) = 0$

$r^2 - 100 = 0.$

$y'' - 100y = 0.$

Problem 3 $y_1 = \sin x^2$ and $y_2 = \cos x^2$ are linearly independent functions, but show that their Wronskian vanishes (is equal to zero) at $x = 0$. Why does this imply that there is no DEQ having both y_1 and y_2 as (global) solutions, of the form $y'' + p_1(x)y' + p_2(x)y = 0$, with both p_1 and p_2 continuous everywhere?

$$W(y_1, y_2) = \begin{vmatrix} \sin x^2 & \cos x^2 \\ 2x\cos x^2 & -2x\sin x^2 \end{vmatrix}$$

$$= -2x\sin^2 x^2 - 2x\cos^2 x^2$$

$$= -2x(\sin^2 x^2 + \cos^2 x^2) = -2x.$$

$-2x$ vanishes at $x = 0.$

"**Why does this imply that there is no differential equation of the form** $y'' + p_1(x)y' + p_2(x)y = 0$, **with both** p_1 **and** p_2 **continuous everywhere**, **having both** y_1 **and** y_2 **as global solutions**?"

In order for y_1 and y_2 to be linearly **independent** solutions of the equation $y'' + p_1y' + p_2y = 0$

(w/p_1 and p_2 both continuous) on an open interval I containing $x = 0$,

the **Wronskian of Solutions Theorem** requires $W \neq 0$ on all of I.