

4.4 Exercises - Solutions

Problem: #1 Determine whether or not the given vectors in \mathbb{R}^4 form a basis for \mathbb{R}^4 .

$$\vec{v}_1 = (2, 0, 0, 0), \quad \vec{v}_2 = (0, 3, 0, 0), \quad \vec{v}_3 = (0, 0, 7, 6), \quad \vec{v}_4 = (0, 0, 4, 5)$$

$$\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 7 & 4 \\ 0 & 0 & 6 & 5 \end{vmatrix} = 2 \cdot 3(35 - 24) = 66 \neq 0.$$

So the four vectors (same number as $\dim(\mathbb{R}^4)$) are **linearly independent**, and hence *do* form a basis for \mathbb{R}^4 .

Problem: #2 Find a basis for the **subspace** of \mathbb{R}^4 which consists of vectors of the form (a, b, c, d) such that $a = 3c$ and $b = 4d$.

Can be written as... $\vec{v} = (3c, 4d, c, d)$

$$= c(3, 0, 1, 0) + d(0, 4, 0, 1).$$

So let: $\vec{v}_1 = (3, 0, 1, 0)$ and $\vec{v}_2 = (0, 4, 0, 1)$.

And a basis is $\{\vec{v}_1, \vec{v}_2\}$.