4.3 Exercises

Problem 1 If possible, express $\vec{w} = (7, 7, 9, 11)$ as a linear combination of

$\vec{v}_1 = (2, 0, 3, 1)$, $\vec{v}_2 = (4, 1, 3, 2)$, $\vec{v}_3 = (1, 3, -1, 3)$. If not, show that it is impossible.

Problem 2 If the following vectors are linearly independent, show it.

Otherwise, find a nontrivial linear combination of them that is equal to the zero vector.

$\vec{v}_1 = (3, 9, 0, 5)$, $\vec{v}_2 = (3, 0, 9, -7)$, $\vec{v}_3 = (4, 7, 5, 0)$

Problem 3 Let's assume the set of vectors $\{\vec{v}_i\}$ are linearly independent. Apply the definition of linear independence to show that the vectors below are also linearly independent.

$\vec{u}_1 = \vec{v}_2 + \vec{v}_3, \qquad \vec{u}_2 = \vec{v}_1 + \vec{v}_3, \qquad \vec{u}_3 = \vec{v}_1 + \vec{v}_2$

Problem 4 **Prove**: If a set S of vectors is linearly dependent and a (finite) set T contains S, then T is also linearly dependent. Assume $S = \{\vec{v}_1, \vec{v}_2 \ldots, \vec{v}_k\}$ and $T = \{\vec{v}_1, \vec{v}_2 \ldots, \vec{v}_k, \ldots, \vec{v}_m\}$, with $m > k$.