

4.2 Exercises

Problem 1 For the following system of equations, find two solution vectors \vec{u} and \vec{v} (a basis) such that the **solution space** is the set of all linear combinations of the form $s\vec{u} + t\vec{v}$.

$$\begin{aligned}x_1 - 4x_2 - 3x_3 - 7x_4 &= 0 \\2x_1 - x_2 + x_3 + 7x_4 &= 0 \\x_1 + 2x_2 + 3x_3 + 11x_4 &= 0\end{aligned}$$

Hint: $\mathbf{A} = \begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix} \xRightarrow{\text{trust me}} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

Problem 2 Reduce the given system to echelon form to find a single solution vector \vec{u} such that the solution space is the set of all scalar multiples of \vec{u} .

$$\begin{aligned}x_1 + 3x_2 + 3x_3 + 3x_4 &= 0, \\2x_1 + 7x_2 + 5x_3 - x_4 &= 0, \\2x_1 + 7x_2 + 4x_3 - 4x_4 &= 0.\end{aligned}$$

Problem 3 Let \mathbf{A} be an $n \times n$ matrix, \vec{b} be a nonzero vector, and \vec{x}_0 be a solution vector to: $\mathbf{A}\vec{x} = \vec{b}$.
Show that \vec{x}_2 is another solution **if and only if** $(\Leftrightarrow) \vec{x}_2 - \vec{x}_0$ is a solution of the homogeneous system $\mathbf{A}\vec{y} = \vec{0}$.

Problem 4 Assume W is the set of all vectors in \mathbb{R}^4 such that $x_1 = 3x_3$ and $x_2 = 4x_4$.
Apply the theorems in this section to determine whether or not W is a subspace of \mathbb{R}^4 .