3.4 Exercises - Solutions 💪

Problem 1 Let $\mathbf{A} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find $\mathbf{B}$ so that $\mathbf{AB} = \mathbf{I} = \mathbf{BA}$: (In other words, find the correct $\mathbf{B}$ so that $\mathbf{A}$ and $\mathbf{B}$ **DO** commute).

- First calculate, and then equate entries on the two sides of the equation $\mathbf{AB} = \mathbf{I}$.
- Then solve the resulting four equations for $a, b, c,$ and d.

The matrix equation $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ldots$

$$\begin{bmatrix} 5a + 7c & 5b + 7d \\ 2a + 3c & 2b + 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$5a + 7c = 1, \quad 2a + 3c = 0, \quad 5b + 7d = 0, \quad 2b + 3d = 1.$

Then we solve for: $a = 3, \quad b = -7, \quad c = -2, \quad d = 5.$

$$\mathbf{B} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}.$$

You can check your work by verifying that $\mathbf{BA} = \mathbf{I}$ as well.

$$\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 15 - 14 & 21 - 21 \\ -10 + 10 & -14 + 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \checkmark$$

Problem 2 Find four different 2×2 matrices $\mathbf{A}$, with each main diagonal element either $+1$ or -1, such that $\mathbf{A}^2 = \mathbf{I}$.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$