3.1 Exercises - Solutions

Problem 1 Use the method of elimination to determine whether the given linear system is **consistent** (has at least one solution) or **inconsistent** (has no solution). If the system is consistent, and if its solution is unique, provide it. Otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$x + 5y + 6z = 3$
$5x + 2y - 10z = 1$
$8x + 17y + 8z = 5$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 3 \\ 5 & 2 & -10 & 1 \\ 8 & 17 & 8 & 5 \end{array}\right] \Rightarrow R_2 + (-5R_1) \text{ and } R_3 + (-8R_1) \Rightarrow \left[\begin{array}{ccc|c} 1 & 5 & 6 & 3 \\ 0 & -23 & -40 & -14 \\ 0 & -23 & -40 & -19 \end{array}\right]$$

$$\Rightarrow R_3 + (-R_2) \Rightarrow \left[\begin{array}{ccc|c} 1 & 5 & 6 & 3 \\ 0 & -23 & -40 & -14 \\ 0 & 0 & 0 & -5 \end{array}\right] \quad !?!?$$

The system is inconsistent, there are no solutions.

Problem 2 Use the method of elimination to determine whether the given linear system is **consistent** (has at least one solution) or **inconsistent** (has no solution). If the system is consistent, and if its solution is unique, provide it. Otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$2x + 2y - 2z = 10$
$3x + y + 3z = 11$
$5z + 4x + y = 14$

$$\left[\begin{array}{ccc|c} 2 & 2 & -2 & 10 \\ 3 & 1 & 3 & 11 \\ 4 & 1 & 5 & 14 \end{array}\right] \Rightarrow \tfrac{1}{2}R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 3 & 1 & 3 & 11 \\ 4 & 1 & 5 & 14 \end{array}\right]$$

$$\Rightarrow R_2 + (-3R_1) \text{ and } R_3 + (-4R_1) \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & -2 & 6 & -4 \\ 0 & -3 & 9 & -6 \end{array}\right]$$

$$\Rightarrow -\tfrac{1}{2}R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & -3 & 9 & -6 \end{array}\right] \Rightarrow R_3 + 3R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & -3 & 2 \end{array}\right]$$

Is there a solution? How do we write it?

We let the "free variable" z from the "free column" be arbitrary: $z \to t$.

It follows that $y = 3t + 2$, and also that:

$$x = -y + t + 5 = -(3t + 2) + t + 5 = -2t + 3.$$

So the infinite solution set is: $(x,y,z) = \big(-2t + 3,\ 3t + 2,\ t\big)$, for every $t \in \mathbb{R}$.

Problem 3 Given: $y'' - 10y' + 21y = 0$, and $y(x) = Ae^{3x} + Be^{7x}$, determine the constants A and B, so as to find a solution of the differential equation that satisfies the initial conditions: $y(0) = 15,\ y'(0) = 13$.

$y(0) = Ae^{3\cdot 0} + Be^{7\cdot 0} = 15$, So, $A + B = 15$.

$y' = 3Ae^{3x} + 7Be^{7x}$ and $13 = 3Ae^{3\cdot 0} + 7Be^{7\cdot 0}$, So, $3A + 7B = 13$.

We could use the symbolic substitution method. Or, using our new matrix technique:

$$\left[\begin{array}{cc|c} 1 & 1 & 15 \\ 3 & 7 & 13 \end{array}\right]$$

$$\Rightarrow R_2 - 3R_1 \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 15 \\ 0 & 4 & -32 \end{array}\right]$$

$$\Rightarrow \tfrac{1}{4}R_2 \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 15 \\ 0 & 1 & -8 \end{array}\right]$$

With $B = -8$ and $A = 15 - B = 15 - (-8) = 23$.

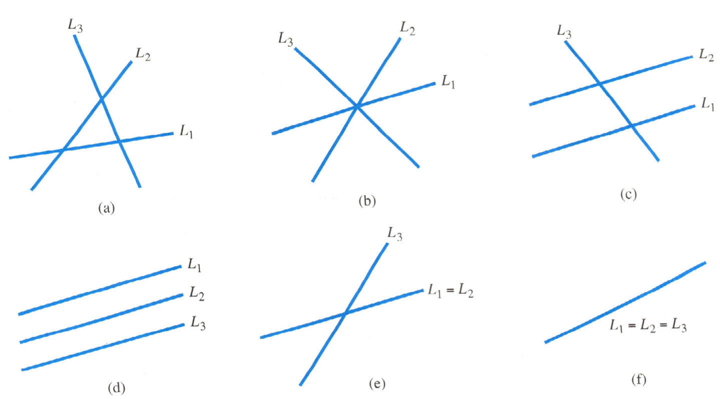
Thus the solution of the differential equation is: $y(x) = 23e^{3x} - 8e^{7x}$.

Problem 4

The linear system:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$
$$a_3x + b_3y = c_3$$

of three equations in two unknowns (x, y) represents three lines L_1, L_2, and L_3 in the xy-plane. The figures below show six possible configurations of these 3 lines. In each case, describe the solution set of the system:



a) no sol. b) 1 sol. c) no sol. d) no sol. e) 1 sol. f) infinite sols.