

2.4 Exercises

Problem 1 Consider the initial value problem: $y' = e^{-y}$, $y(0) = 0$, with exact solution: $y(x) = \ln(x + 1)$.

Apply Euler's method to approximate this soln on the interval $[0, \frac{1}{2}]$; first w/step size $h_1 = 0.25$, then w/step size $h_2 = 0.1$. Compare the values of the approximations at $x = \frac{1}{2}$ w/the value $y(\frac{1}{2})$ of the actual soln ($y(\frac{1}{2}) = \ln(\frac{1}{2} + 1) \approx 0.405$).

Problem 2 Consider the initial value problem: $y' = 2xy^2$, $y(0) = 1$; Exact solution: $y(x) = \frac{1}{1-x^2}$.
Apply Euler's method to approximate to this solution on the interval $[0, \frac{1}{2}]$;
First with step size $h_1 = 0.25$, then with step size $h_2 = 0.1$. Compare with the exact solution above.