

2.2 Exercises

Problem 1. Which of the following is true about the solution of the initial value problem:

$$\frac{dx}{dt} = 6x - x^2 - 8, \quad x(0) = 1$$

A) $\lim_{t \rightarrow \infty} x(t) = -2$

B) $\lim_{t \rightarrow -\infty} x(t) = -2$

C) $\lim_{t \rightarrow \infty} x(t) = 4$

D) $\lim_{t \rightarrow -\infty} x(t) = 4$

E) $\lim_{t \rightarrow -\infty} x(t) = \infty$

F) $\lim_{t \rightarrow \infty} x(t) = -\infty$

Problem 2. A ski jumper launches from rest down a long ramp. Her acceleration is proportional to the difference between 40 m/s (her theoretical maximum speed due to air resistance and ramp slope) and her current velocity. After 10 seconds, her speed is measured to be 20 m/s. How long will it take for the jumper to reach 35 m/s?

Problem 3 First solve $f(x) = 0$ to find the critical points of the autonomous DEQ $\frac{dx}{dt} = f(x) = 7x - x^2 - 10$. Then analyze the sign of $f(x)$ to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the DEQ. Next, solve the DEQ explicitly for $x(t)$ in terms of t . Finally, use either the exact solution or a computer-generated slope field to sketch typical solution curves for the given DEQ, and verify visually the stability of each critical point.