
Probability Theory: Activity 18

Name: _____

To study the prevalence of a disease in a several cities of interest within a certain county, we pick a city at random, then pick a random sample of n people from that city. Let p be the proportion of diseased people in the chosen city, and let X be the number of diseased people in the sample. Since every city has a different disease proportion, p is a random variable. Suppose that $p \sim Unif(0, 1)$. Also, assume that, given p , assume $X|p$ follows a binomial(n, p).

Find $E(X)$ and $Var(X)$.

1. Use the properties of the binomial to find $E(X|p)$ and $Var(X|p)$.
2. Use the fact that $p \sim Unif(0,1)$ to find $E(p)$ and $Var(p)$.
3. Find $E(X)$ using Adams law: $E(X) = E(E(X|p))$.
4. Find $E(Var(X|p))$.
5. Find $Var(E(X|p))$.
6. Find $Var(X)$ using Eve's law: $Var(X) = E(Var(X|p)) + Var(E(X|p))$.