Probability Theory: Activity 14 Solutions

1. Let X and Y be discrete random variables with the following joint PF:

$$f(x,y)=\begin{cases}\frac{1}{7}(x-y)^2 & \text{for } x=0,1,2 \text{ and } y=0,1\\ 0 & \text{otherwise}\end{cases}$$

(a) Find $P(X=1,Y=0)$.

$$P(X=1,Y=0)=f(1,0)=\frac{1}{7}(1-0)^2=\frac{1}{7}$$

(b) Find $P(X\le 1,Y=0)$

$$Pr(X\le 1,Y=0)=P(X=0,Y=0)+P(X=1,Y=0)=0+\frac{1}{7}=\frac{1}{7}.$$

(c) Find the marginal PF of X:

$$f_X(x)=\sum_y \frac{1}{7}(x-y)^2=\frac{1}{7}\left(x^2+(x-1)^2\right)$$

2. Let X and Y be continuous random variables with the following joint PF:

$$f(x,y)=\begin{cases}\frac{3}{2}y^2 & \text{for } 0\le x\le 2 \text{ and } 0\le y\le 1\\ 0 & \text{otherwise}\end{cases}$$

(a) Find $P(X<1,Y\ge\frac{1}{2})$

$$P(X<1,Y\ge\frac{1}{2})=\int_{\frac{1}{2}}^{1}\int_0^1\frac{3}{2}y^2dxdy=\int_{\frac{1}{2}}^{1}\frac{3}{2}y^2x\Big|_0^1dy$$

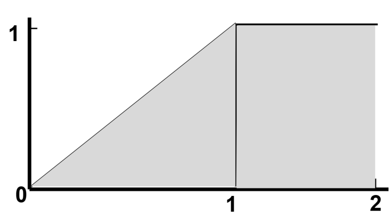
$$=\int_{\frac{1}{2}}^{1}\frac{3}{2}y^2dy=\frac{1}{2}y^3\Big|_{\frac{1}{2}}^{1}=\frac{1}{2}-\frac{1}{16}=\frac{7}{16}$$

(b) Find $P(X>Y)$

$$P(X>Y)=\int_0^1\int_y^2\frac{3}{2}y^2dxdy=\int_0^1\frac{3}{2}y^2x\Big|_y^2dy=\int_0^1\frac{3}{2}(2y^2-y^3)dy$$

$$=\frac{3}{2}(\frac{2}{3}y^3-\frac{1}{4}y^4)\Big|_0^1=\frac{3}{2}(\frac{2}{3}-\frac{1}{4})=\frac{5}{8}$$

If you switch order of integration, things get more difficult, due to the region you are integrating over.



So, we must evaluate:

$$P(X>Y)=\int_0^1\int_0^x\frac{3}{2}y^2dydx+\int_1^2\int_0^1\frac{3}{2}y^2dxdy$$

But you should get the same answer.

(c) Find the marginal PF of Y:

$$f_X(x) = \int_0^2 \frac{3}{2}y^2 dx = \frac{3}{4}y^2x^2|_0^2 = 3y^2$$

3. Suppose that X and Y are continuous rvs with the following joint CDF on the interval (0,2) for both X and Y:

$$F(x,y) = \frac{1}{16}xy(x+y) \qquad 0 < y < 2, \quad 0 < x < 2$$

(a) Find $F_x(x)$

$$F_x(x) = \lim_{y->\infty} F(x,y) = F(x,2) = \frac{1}{16}2x(x+2) = \frac{1}{8}x(x+2) \quad 0 < x < 2$$

(b) Find the joint PF.

$$\frac{\partial^2}{\partial x \partial y}F(x,y) = \frac{\partial^2}{\partial x \partial y}\frac{1}{16}(x^2y + xy^2)$$

$$\frac{\partial}{\partial y}\frac{1}{16}(2xy + y^2) = \frac{1}{16}(2x + 2y)$$

$$f(x,y) = \begin{cases} \frac{x+y}{8} & 0 < x < 2, \quad 0 < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

4. Let X be a discrete random variable and Y be continuous random variable with the following joint PF:

$$f(x,y) = \begin{cases} cy^x & \text{for } x = 0,1,2 \text{ and } 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find c.

$$1 = \int_0^2 \sum_{x=0}^{2} cy^x dy = \int_0^2 c(1 + y + y^2)dy = c(y + \frac{1}{2}y^2 + \frac{1}{3}y^3)|_0^2 = c(4 + \frac{8}{3})$$

$$c = \frac{3}{20}$$