Probability Theory: Activity 12 Solutions

1. Let X be a discrete random variable with the following PF:

$$f_x(x) = \begin{cases} \frac{x^2}{10} & \text{for } x = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the least median of X.

$$F(-2) = \frac{4}{10}$$

$$F(-1) = P(X \leq -1) = \frac{4}{10} + \frac{1}{10} = \frac{5}{10} = 0.5$$

   The mininum value such that F(c) is at least 0.5 is -1, so -1 is the least median.

   (b) Find the mode(s) of X.

   Both -2 and 2 have a probability of $\frac{4}{10}$, which is the max probability of any value. So -2 and 2 are both modes.

   (c) Find the first moment of X.

$$E(X) = \sum_{x=-2}^{2} x\frac{x^2}{10} = -2\frac{4}{10} + -1\frac{1}{10} + 0(0) + 1\frac{1}{10} + 2\frac{4}{10} = 0$$

   (d) Find the 3rd central moment of X.

$$E((X - E(X))^3) = E((X - 0)^3) = E(X^3) = \sum_{x=-2}^{2} x^3\frac{x^2}{10}$$

$$= (-2)^2\frac{4}{10} + (-1)^2\frac{1}{10} + 0^3(0) + 1^3\frac{1}{10} + 2^3\frac{4}{10} = 0$$

2. Let X be a continuous random variable with following PF:

$$f_x(x) = \begin{cases} \frac{3}{4}(2x - x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find E(X).

$$E(X) = \int_0^1 xf(x)dx = \int_0^1 \frac{3}{4}(2x^2 - x^3) = \frac{3}{4}(\frac{2}{3}x^3 - \frac{1}{4}x^4)|_0^1 = \frac{5}{16}$$

   (b) Find the third moment of X.

$$E(X^3) = \int_0^1 x^3f(x)dx = \int_0^1 \frac{3}{4}(2x^4 - x^5) = \frac{3}{4}(\frac{2}{5}x^5 - \frac{1}{6}x^6)|_0^1 = \frac{5}{16} = \frac{7}{40}$$

   (c) Find the mode of X.

   To find the mode, we take the derivative of f(x) and set it equal to 0.

$$\frac{d}{dx}f(x) = \frac{3}{4}(2 - 2x)$$

$$0 = \frac{3}{4}(2 - 2x)$$

$$x = 1$$

   At $x = 1$, $f(1) = \frac{3}{4}$, which is the max value it takes on (since $f'(x) = \frac{3}{4}(2 - 2x)$ is positive on $0 < x < 1$).