Probability Theory: Activity 11 Solutions

1. Suppose that X follows a Normal distribution with a mean of 3 and a variance of 9.

   (a) What is the probability of observing X less than 4.5?

$$\begin{aligned} P(X < 4.5) &= P\left(\frac{X-3}{3} < \frac{4.5-3}{3}\right) \\ &= P(Z < .5) \\ &= \Phi(0.5) = 0.6915 \end{aligned}$$

   (b) What is the probability of observing X greater than 0?

$$\begin{aligned} P(X > 0) &= P\left(\frac{X-3}{3} > \frac{0-3}{3}\right) \\ &= P(Z > -1) \\ &= 1 - P(Z < -1) \\ &= 1 - \Phi(-1) \\ &= 1 - (1 - \phi(1)) = 0.8413 \end{aligned}$$

   (c) Give a value c such that the probability of observing a value of X less than c is 0.99.

$$\begin{aligned} P(X < c) &= 0.99 \\ P\left(\frac{X-3}{3} < \frac{c-3}{3}\right) &= 0.99 \\ P\left(Z < \frac{c-3}{3}\right) &= 0.99 \\ P(Z < 2.33) &= 0.99 \end{aligned}$$

   The last step comes from looking up the value of Z such that we get a probablitiy of 0.99. Now we can just solve for c:

$$\frac{c-3}{3} = 2.33$$

   $c = 9.99$.

2. Suppose the wait time at the doctors is 20 minutes between patients on average.

   (a) What is the probability of waiting fewer than 30 minutes?
   This follows an exponential distribution with a mean of 20, so $\lambda = \frac{1}{20}$. So we have:

   $$P(X < 30) = 1 - e^{-30/20} = 0.7768.$$

   (b) What is the probability of waiting another 10 minutes if you've already waiting 20 minutes?
   We can use the memoryless property.

   $$Pr(X > 30|X > 20) = P(X > 10) = e^{-10/20} = 0.606.$$

   (c) What is the probability of the doctor only seeing 2 patients in an hour?
   Since the doctor sees a patient every 20 minutes, he sees 3 patients in an hour on average. So this follows a Poisson distribution with a mean of 3.

   $$P(Y = 2) = \frac{3^2 e^{-3}}{2!} = 0.224$$