Probability Theory: Activity 9 Solutions

1. Let X be a random variable with the following PF:

$$f_x(x) = \begin{cases} \frac{x^2}{10} & \text{for } x = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

   Find $\text{Var}(X)$.

   $Var(X) = E(X^2) - E(X)^2$. We know from activity 8 that the mean is 0 and that $E(X^2) = 3.4$. So the variance is $3.4 - 0^2 = 3.4$.

2. Let X have variance 5 and Y have variance 4. Assume that X and Y are independent. Find the following:

   (a) $\text{Var}(2X + 2)$

$$\text{Var}(2X + 2) = 4\text{Var}(X) = 20.$$

   (b) $\text{Var}(2X + 3Y + 1)$.

$$\text{Var}(2X + 3Y + 1) = 3\text{Var}(X) + 9\text{Var}(Y) = 15 + 36 = 51.$$

   (c) $\text{SD}(Y)$.

   The standard deviation is the square root of the variance, so is just $\sqrt(4) = 2$.

3. Suppose you are checking the widgets at a factory where 10% of all widgets are known to be defective.

   (a) You take a lot of 100 widgets and check for defective ones. Let X be the number of defective items. What is the mean and variance of X?

   This follows a binomial distribution with $p = 0.1$ and $n = 10$. The mean of a binomial is $np = 100(0.1) = 10$. The variance is $np(1 - p) = 100(0.1)(0.9) = 9$.

   (b) You stand at the conveyer belt of widgets and examine them one by one. What is the probability that the 6th one is the first one you find to be defective?

   This follows a geometric distribution with $p = 0.10$. We want to find 5 working ones before we find the defective one. So we want:

$$P(X = 5) = (1 - p)^5 p = 0.9^5(0.1) = 0.059.$$

   (c) You stand at the conveyer belt of widgets and examine them one by one. What is the average number of widgets you'll expect to examine before you find the defective one?

   This still follows a geometric distribution. The mean of the geometric is q/p, so it is $(1-0.1)/0.1 = 0.9/0.1 = 9$. We will expect to examine 9 non-defective items before our first defective item.

   (d) You stand at the conveyer belt of widgets and examine them one by one. What is the probability that you will have looked at 15 working widgets before you find the 2nd defective widget?

   This follows the negative binomial distribution with $r = 2$ and $p = 0.1$. We want:

$$P(X = 15) = \binom{16}{1}(1 - 0.1)^{15} 0.1^2 = 0.033.$$

   (e) Suppose you find defective widgets at the rate of 5 per hour on average. Let X be the number of the defective widgets you find in the next hour. What is the probability that X=6?

   We are now looking at things in terms of a rate, so we want to use the Poisson with a mean of 5. We find:

$$P(X = 6) = \frac{e^{-5} 5^6}{6!} = 0.146.$$