**Probability Theory: Activity 7 Solutions**

1. Let $X$ be a random variable with the following PF:

$$f(x) = \begin{cases} \frac{x^2}{10} & \text{for } x = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that this is a valid PF.

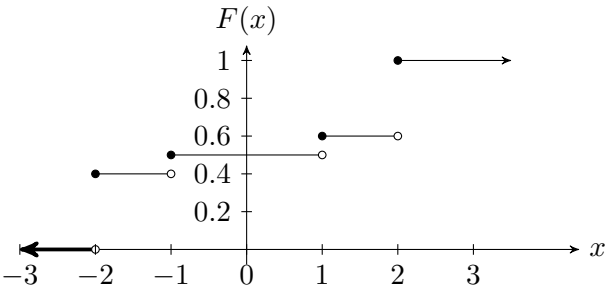
To be a valid PF., this must sum to 1 and be positive for all values. Since the funciton is $x^2$, we know it is positive for all values. To show it sums to 1:

$$\sum_{x=-2}^{2} \frac{x^2}{10} = \frac{4}{10} + \frac{1}{10} + 0 + \frac{1}{10} + \frac{4}{10} = \frac{10}{10} = 1.$$

(b) Find the CDF of X. Plot it.

The CDF of X is:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.4 & -2 \le x < -1 \\ 0.5 & -1 \le x < 1 \\ 0.6 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$




(c) What is $f(-1)$? What is $F(-1)$?

$$f(-1) = \frac{(-1)^2}{10} = 0.1.$$

$$F(-1) = P(X \le -1) = f(-2) + f(-1) = 0.4 + 0.1 = 0.5.$$

(d) Let $Y = X^2$. What is the PF of Y?

If $Y = X^2$, it can take only take on the values 0, 1, and 4.

$$f_Y(x) = \begin{cases} \frac{y}{5} & \text{for } y = 1, 4 \\ 0 & \text{otherwise} \end{cases}$$

2. Consider an experiment in which we roll two dice. Let $X$ be the sum of the two dice. Let $Y$ be the absolute difference between the two dice.

   (a) Find $f_x(10)$. (Eg, there are 36 possible outcomes to an experiment that rolls two dice. How many sum to 10?)

   There are a total of 3 ways to get a 10: (4,6), (5,5) and (6,4). So the $f_x(10) = P(X = 10) = \frac{3}{36} = \frac{1}{12}$.

   (b) Find $f_y(2)$. (Eg, there are 36 possible outcomes to an experiment that rolls two dice. How many have a difference of 2?)

   There are a total of 8 ways to get a difference of 2: (1,3), (2, 4) (3, 1) (3, 5) (4,2) (4,6) (5, 3) (6, 4). So we have $f_y(2) = P(Y = 2) = \frac{8}{36} = \frac{2}{9}$.

   (c) Find $f_{x,y,}(x = 10, y = 2)$. (How many outcomes both sum to 10 and have a difference of 2?)

   There are 3 total outcomes that sum to 10, but only 2 have a difference of 2: (4,6) and (6,4). So $f_{x,y}(x = 10, y = 2) = \frac{2}{36} = \frac{1}{18}$.

   (d) Are $X$ and $Y$ independent? Why or why not?

   We know that $f_{x,y}(x = 10, y = 2) = \frac{2}{36} = \frac{1}{18}$. We can also find $f_x(10)f_y(2) = \frac{1}{12} \times \frac{2}{9} = \frac{1}{54}$. These are not equal, therefore, $X$ and $Y$ are not independent.