Probability Theory: Activity 6 Solutions

1. Suppose you and your friend have entered the dorm room lottery. You find out you're going to be on the third floor, but you have an equal chance of getting any room from 301 to 335. Let X be the room number you get.

   (a) What is the name of the distribution of X?

   This is the uniform distribution on the integers from 301 to 335.

   (b) What is the probability function of X?

   $$f(x) = \begin{cases} \frac{1}{35} & \text{if } x = 301, 302, \ldots, 335 \\ 0 & \text{otherwise} \end{cases}$$

   (c) You want an even room, because then you'll have a better view. What is the probability of an even room?

   There are 16 even rooms. So we have $\Pr(X \text{ is even}) = 16/35$.

2. A baseball team has a record of 88-66. Assume the probability of winning any particular game is constant, and each game is independent. Let X be 1 if they win the next game and 0 if they lose.

   (a) What is the name of the distribution function of X?

   This is a Bernoulli distribution with $p = 88/154 = 8/14$.

   (b) What is the probability of winning the next game?

   We can estimate it as the number of wins over the total number of games: $\frac{88}{88+66} = 8/14$.

   (c) What is the PMF of X?

   $$f(x) = \begin{cases} 6/14 & \text{if } x = 0 \\ 8/14 & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

3. The baseball team from problem 2 plays a series of 10 games. Let X be the number of games they win.

   (a) What is the name of the distribution of X?

   X is distributed as a Binomial, with $n = 10$ and $p = 8/14$.

   (b) What is the PMF of X?

   $$f(x) = \begin{cases} \binom{10}{x} \left(\frac{8}{16}\right)^x \left(1 - \frac{8}{16}\right)^{10-x} & \text{if } x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

   (c) What is the probability of them winning 6 games?

   $$f(6) = \binom{10}{6} \left(\frac{8}{16}\right)^6 \left(1 - \frac{8}{16}\right)^{10-6} = .2466$$

4. A baseball player has a 0.300 batting average, meaning that he has a 30% chance of hitting the ball for each at bat. He is at bat 5 times in a game. Let X be the number of hits he makes.

   (a) What is the name of the distribution of X?

   The distribution of X is binomial, with $n = 5$ and $p = .3$.

(b) What is the probability function of X?

$$f(x) = \begin{cases} \binom{5}{x}(.3)^x(1-.3)^{5-x} & \text{if } x = 0, 1, \ldots, 5 \\ 0 & \text{otherwise} \end{cases}$$

(c) What is the probability of him making 3 hits?

$$f(3) = \binom{5}{3}(.3)^3(.7)^2 = .1323$$

5. The students in your class are randomly numbered 2-9 (you have number 1). The teacher is going to randomly select the number X of your partner from the list.

(a) What is the name of the distribution of X?

This is the uniform distribution on the integers from 2 to 9.

(b) What is the probability function of X?

$$f(x) = \begin{cases} \frac{1}{8} & \text{if } x = 2, \ldots, 9 \\ 0 & \text{otherwise} \end{cases}$$

(c) Your friends have numbers 5, 6, and 7. What's the probability that you're going to have one of your friends?

$$f(5) + f(6) + f(7) = \frac{3}{8}$$

6. Your teacher is going to select 5 students from your class of 23 students to present problems at the board. In the class, 10 are women and 13 are men. Let X be the number of women she selects.

(a) What is the name of the distribution of X?

This is selecting 5 people without replacement from a pool of 23, so this follows a hypergeometric distribution. We have 10 possible successes, 13 possible failures, and 5 draws.

(b) What is the probability function of X?

In this case, a = 10, b=13, and n=5. So we have:

$$f(x) = \begin{cases} \frac{\binom{10}{x}\binom{13}{5-x}}{\binom{23}{5}} & \text{if } x = 0, \ldots, 5 \\ 0 & \text{otherwise} \end{cases}$$

(c) What is the probability that 3 of the students who present are women?

$$f(3) = \frac{\binom{10}{3}\binom{13}{2}}{\binom{23}{5}} = 0.278$$