Probability Theory: Activity 5b Solutions

1. A tech company is developing a new smartphone, and its success depends on multiple independent components working as intended. Specifically:

   The probability that the phone's processor works without failure is 0.95. The probability that the phone's battery meets performance standards is 0.9. The probability that the phone's screen passes quality control is 0.85. Assume these components function independently of one another.

   (a) What is the probability that a randomly selected smartphone has all three components working as intended?

   Solution: We are given the probabilities:
   Processor works: P(A)=0.95
   Battery works: P(B)=0.9
   Screen works: P(C)=0.85
   Since the events are independent, the probability of all three working is the product of their individual probabilities: $P(A \cap B \cap C) = P(A)P(B)P(C) = (0.95)(0.9)(0.85) = 0.72675$.

   (b) If the company produces 10,000 smartphones, how many would you expect to have all three components working?

   Solution: Expected smartphones $= 0.72675 \times 10,000 = 7,267.5$.

   (c) What is the probability that at least one of the components fails?

   Solution: To find this, we use the complement rule: $P(\text{At least one fails}) = 1 - P(\text{All three work}) = 1 - 0.72675 = 0.27325$.

2. Is it possible that an event is independent of itself? If so, when is this the case?

   Solution: Yes, observe that if A is indep of itself, then $P(A) = P(A \cap A) = P(A)P(A) = P(A)^2$. So either $P(A) = 0$ or $P(A) = 1$.