Probability Theory: Activity 5aa Solutions

A good friend of yours is convinced that his presence is required for his favorite basketball team to win. He shows you the following chart:

Overall	Friend	No Friend	Total
Win	13	9	22
Loss	10	22	32
Total	23	31	54

"Look!" your friend says, "when I am at the game, we win more than half our games. But when I'm not at the game, we win less than half. Clearly I need to attend all the game from now on!"

1. Find the probability of a win, given your friend is present: $P(W|F)$.

   $P(W|F) = 13/23 \approx 0.565$

2. Find the probability of a win, given your friend is not present:$P(W|F^c)$.

   $P(W|F^c) = 9/31 \approx 0.290$

3. Find the probability of a win: $P(W)$.

   $P(W) = 22/54 \approx 0.407$

4. Do you think that the probability of winning is independent of your friend's presence?

   They are not independent, because $P(W|F)$ does not equal $P(W)$.

   Well, this looks awfully strange. You can't believe that your friend really makes a differencsitting in the stands. You recall that your (otherwise not very good team) has one really strong amazing star player who your friend really loves, but that sometimes that player is hurt or needs a rest, and so doesn't play. Maybe this has something to do with it. So you ask your friend, how many games did he attend and see that star? He gives you the following tables:

Overall	Friend	No Friend	Total
Win	12	4	16
Loss	6	2	18
Total	18	6	24
With Star			

Overall	Friend	No Friend	Total
Win	1	5	6
Loss	4	20	24
Total	5	25	30
Without Star			

5. Now, you wonder, what is the probability of a win given the star and your friend's presence? That is, what is $P(W|F \cap S)$?

   $12/18 = 2/3$

6. What is the probability of a win given the star and your friend's absence? That is, what is $P(W|F^c \cap S)$?

   $4/6 = 2/3$

7. Now, what is the probability of a win given the star? What is $P(W|S)$?

   $16/24 = 2/3$

8. Similarly, what's the probability of winning given that the team doesn't have the star player and your friend is absent? That is, what is $P(W|S^c \cap F^c)$?

   $5/25 = 1/5$

9. What is the probability of winning given that your friend is present, but the team doesn't have the star player? That is, what is $P(W|S^c \cap F)$?

   $1/5$

10. What is the probability of winning given that the star player is absent? That is, what is $P(W|S^c)$

    $6/30 = 1/5$

11. What do you think is causing your friend to think that his presence effects the team?

    Your friend attends the same games that the star player plays at. That is, the star player is affecting the wins AND your friend's presence. Once we know the star player's games, we can see that $P(W|S \cap F) = P(W|S)$, that is, given the star player, "wins" and "your friend's presence" are conditionally independent.