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## Probability Theory: Activity 5 Solutions

1. Suppose that 5 percent of men and 0.25 percent of women are colorblind. A colorblind person is chosen at random.

(a) Let  $C$  be the event that a person is colorblind and  $M$  be the event that a person is male. Write the two probabilities above in terms of these events.

Since 5% of men are colorblind,  $\Pr(C|M) = 0.05$ . For women, we have  $\Pr(C|M^c) = 0.0025$ .

(b) Assume that there are an equal number of males and females. What is the probability that your randomly selected person is male?

We use Bayes' Theorem. We have a colorblind person, so we want  $\Pr(M|C)$ . This is:

$$\begin{aligned}\Pr(M|C) &= \frac{\Pr(C|M) \Pr(M)}{\Pr(C|M) \Pr(M) + \Pr(C|M^c) \Pr(M^c)} \\ &= \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.0025)(0.05)} \\ &= 0.9523\end{aligned}$$

(c) Assume that there are twice as many males as females. Now what is the probability that your randomly selected person is male?

If there are twice as many males as females, then  $\Pr(M) = 2/3$  and  $\Pr(M^c) = 1/3$ . So we have:

$$\begin{aligned}\Pr(M|C) &= \frac{\Pr(C|M) \Pr(M)}{\Pr(C|M) \Pr(M) + \Pr(C|M^c) \Pr(M^c)} \\ &= \frac{(0.05)(2/3)}{(0.05)(2/3) + (0.0025)(1/3)} \\ &= 0.9756.\end{aligned}$$

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2. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five year period. Calculate the probability that the participant was a heavy smoker.

The problem gives us  $\Pr(H) = 0.2$ ,  $\Pr(L) = 0.3$ , and  $\Pr(N) = 0.5$ . If we let  $\Pr(D|L) = x$ , then it also gives us that  $\Pr(D|N) = 0.5x$  and  $\Pr(D|H) = 2x$ . We want  $\Pr(H|D)$ , so we need to use Bayes' Theorem.

$$\begin{aligned}\Pr(H|D) &= \frac{\Pr(D|H) \Pr(H)}{\Pr(D|H) \Pr(H) + \Pr(D|L) \Pr(L) + \Pr(D|N) \Pr(N)} \\ &= \frac{2x(0.2)}{2x(0.2) + x(0.3) + 0.5x(0.5)} \\ &= \frac{0.4x}{0.95x} \\ &= \frac{0.4}{0.95} \approx 0.421\end{aligned}$$