Probability Theory: Activity 3 Solutions

1. A coded message from a CIA operative to his Russian KGB counterpart is to be sent in the form Q4ET, where the first and last elements of each block are consonants, the second is an integer 1 to 9, and the third is one of the six vowels. If all blocks are equally likely, what is the probability that a randomly intercepted block begins and ends with the same letter?

   Employing the naive definition of probability, we calculate the total number of possible blocks that can be found with the multiplication rule: $20 \times 9 \times 6 \times 20$. For the first and last to be the same, we again use the multiplication rule: $20 \times 9 \times 6 \times 1$, where the last one is a 1 because that term is forced to match the first. So the probability is:

$$\frac{20 \times 9 \times 6 \times 1}{20 \times 9 \times 6 \times 20} = \frac{1}{20}$$

2. The board of a large corporation has seven members - 4 right-handed and 3 left-handed - willing to be nominated for office. If they need a president, vice president, and treasurer, what is the probability that the treasurer and president are left-handed and the vice president is right-handed?

   We need to select three people, and order matters here, because we have a president, treasurer, and vice president (so we can select three people, and all six configurations of them as president, treasurer and vice president all count as different arrangements, which is what we mean by saying order matters). So we use the permutation $P_{7,3} = 7 \times 6 \times 5$. To have left-handed treasurer and president, we select two from the left-handed members, one for president and one for treasurer, so again order matters (that is, Bob as President and Andy as Treasurer is a different arrangement than Andy as President and Bob as Treasurer), so we have: $P_{3,2} = 3 \times 2$. Then we select a right-handed member for vice president, and there a four choices for that. So we have the probability: $\frac{3 \times 2 \times 4}{7 \times 6 \times 5}$

3. A pastry in a vending machine cost 85 cents. You pay for it with two (identical) quarters, three (identical) dimes, and one nickel. If you put the coins in randomly, what's the probability that you put in the two quarters first?

   There are 6! ways to order the six coins, but as the two quarters are indistinguishable and the three dimes also, we need to divide by 2! and 3! to get the total number of ways to put the coins in the machine. Once we've put in the two quarters first, there are 4! ways to put in the remaining four coins, but the three dimes are not distinct, and so we divide by 3!. So we have the probability: $\frac{\frac{4!}{3!1!}}{\frac{6!}{2!3!1!}}$

4. There are 16 basketball players on a team, six guards, seven forwards, and three centers. They carpool to a game, randomly getting in two vans, with 9 of the players in one van and 7 in the other. What's the prob that the 9 person van has 3 guards, 4 forwards, and 2 centers?

   We first pick 9 players out of the 16 to get in the 9 person van. Order doesn't matter, so we use combinations. So we have $\binom{16}{9}$ ways to pick 9 players for the first van. If we want to choose them such that we have 3 guards, 4 forwards, and 2 centers, then we first choose 3 guards of the 6, 4 forwards of the 7, and 2 centers of the 3, so we have $\binom{6}{3}\binom{7}{4}\binom{3}{2}$.

$$\frac{\binom{6}{3}\binom{7}{4}\binom{3}{2}}{\binom{16}{9}}$$