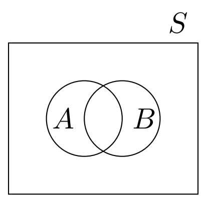
Probability Theory: Activity 2 Solutions

1. A total of 28% of American males smoke cigarettes, 7% smoke cigars, and 5% smoke both cigars and cigarettes.

   (a) Label each of these probabilities on the diagram below:

   Let A = Cigarettes and B= Cigars. Then the A circle = 0.28, the B circle is 0.07, and the space in the middle is 0.05.

   (b) What percentage of males smoke neither cigars nor cigarettes?

   Neither cigars nor cigarettes is Not Cigarettes (A^c) and ($\cap$) Not Cigars (B^c). So we have:

$$\begin{aligned} P(A^c \cap B^c) &= 1 - P((A^c \cap B^c)^c) \text{(By complement rule)} \\ &= 1 - P(A \cup B) \text{(by De Morgan's Laws)} \\ &= 1 - (P(A) + P(B) - P(A \cap B)) \text{(by probability of a union)} \\ &= 1 - (0.28 + 0.07 - 0.05) \\ &= 1 - 0.3 = 0.7 \end{aligned}$$

   (c) What percentage smoke cigars but not cigarettes?

   Smoke Cigars is B and not cigarettes is A^c, so we are looking for $B \cap A^c$. Since we know 7% smoke cigars but 5% also smoke cigarettes, the remainding 2% must only smoke cigars. We can do this algebraically if we recall that A and A^c are a partition:

$$\begin{aligned} P(B) &= P((B \cap A) \cup (B \cap A^c)) \text{(by breaking up B using a partition)} \\ P(B) &= P(B \cap A) + P(B \cap A^c) \text{(Prob of disjoint unions can be summed)} \\ 0.07 &= 0.05 + P(B \cap A^c) \\ P(B \cap A^c) &= 0.02 \end{aligned}$$

2. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. These classes are open to any of the 100 students in the school. There are are 28 students in Spanish, 26 in French, and 16 in German. Also, there are 12 that are in both Spanish and French, 4 in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

   (a) If a student is chosen randomly, what is the probability that they are not in any of these classes?

   We want $F^c \cap G^c \cap S^c$, which is the set of students not in any class. It is easier (by De Morgan's Laws) to find $(F^c \cap G^c \cap S^c)^c = F \cup G \cup S$. To find this union, we extend the union formula for 2 sets that we already developed:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

   We first add the full sets A, B, and C; then we subtract out the intersections, beacuse they've each been added twice. But the triple intersection in the middle has now been added in 3 times (for each A, B, and C) and subtracted 3 times (for each intersection), and so we need to add it back in. So we have:

   $$P(A \cup B \cup C) = 0.28 + 0.26 + 0.16 - 0.12 - 0.04 - 0.06 + 0.02 = 0.5$$

   So the probability of a student not taking any language classes is 1-0.5 = 0.5.

   (b) If a student is chosen randomly, what is the probability that they are taking exactly one language class?

   We have 2 students who are taking three classes. We have 12 in both Spanish and French, but 2 of them are in all 3, so just 10 (12-2) are in only Spanish and French. Similary, 2 (4-2) are in only Spanish and German and 4 (6-2) are in only French and German. We know 50 of the students are in at least one class. 2+10+2+4 = 18 are in more than one class. So 50-18 = 32 are in exactly one language class.