Probability Theory: Activity 1 Solutions

Consider rolling two dice (each a standard 6 sided die) as an experiment, and then you sum them.

1. What are all the possible outcomes, the sample space S?

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

2. What are the elements of the set for the event that the sum is strictly less than 8? Assign this to be set A.

$$A = \{2, 3, 4, 5, 6, 7\}$$

3. What are the elements of the set for the event that the sum is strictly more than 3 but only one digit? Assign this to be set B.

$$B = \{4, 5, 6, 7, 8, 9\}$$

4. What are the elements for the event that the sum is even? Assign this set to be C

$$C = \{2, 4, 6, 8, 10, 12\}$$

Find the following:

1. $A \cup B$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

2. $A \cap B$

$$A \cap B = \{4, 5, 6, 7\}$$

3. A^c

$$A^c = \{8, 9, 10, 11, 12\}$$

4. $(A \cup B)^c$

$$(A \cup B)^c = \{10, 11, 12\}$$

5. $A^c \cap B^c$

$$A^c \cap B^c = \{10, 11, 12\}$$

6. Consider the answers to the previous two questions. Can you find $A^c \cap C^c$ without finding A^c and C^c first?

$$\text{In general, for any } A \text{ and } B\text{, it is true that } (A \cup B)^c = A^c \cap B^c \text{ and } (A \cap B)^c = A^c \cup B^c$$

$$\text{So we can first find } (A \cup C)\text{, and then find the complement: } A^c \cap C^c = (A \cup C)^c = \{9, 11\}$$

7. $C \cup C^c$

$$C \cup C^c = S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

8. $(A \cap C) \cup (A \cap C^c)$

$$(A \cap C) \cup (A \cap C^c) = \{2, 3, 4, 5, 6, 7\} = A$$

9. $(A \cap B) \cup (A \cap B^c)$

$$(A \cap B) \cup (A \cap B^c) = \{2, 3, 4, 5, 6, 7\} = A$$

10. For any other set D, what do you think $(A \cap D) \cup (A \cap D^c)$ is?

$$(A \cap D) \cup (A \cap D^c) = A$$

11. Consider three sets D_1, D_2, D_3 such that they are all disjoint and $D_1 \cup D_2 \cup D_3 = S$. What do you think $(A \cap D_1) \cup (A \cap D_2) \cup (A \cap D_3)$ is? Explain why.

$$(A \cap D_1) \cup (A \cap D_2) \cup (A \cap D_3) = A.$$

This is because if D_1, D_2, and D_3 are all disjoint and cover S, then they must also cover A, so that we can divide A into three pieces whose union is A.