Introduction to Statistics

Activity 9.4d **SOLUTIONS**

Hypothesis Testing- Baseball "Big Bang"

A reader sent a letter to the "Ask Marilyn" column in *Parade* magazine to say that in three-quarters of all baseball games, the winning team scores more runs in one inning than the losing team scores in the entire game. (This phenomenon is known as a "big bang.") Marilyn responded that this proportion seemed too high to be believable. Let p be the proportion of all major-league baseball games in which a big bang occurs.

a) Restate the assertion as the null hypothesis, in symbols and in words.

The null hypothesis is that the proportion of all major-league baseball games that contain a big bang is three-fourths. In symbols, the null hypothesis is $H_0: \pi = .75$.

b) Given Marilyn's conjecture, state the alternative hypothesis, in symbols and in words.

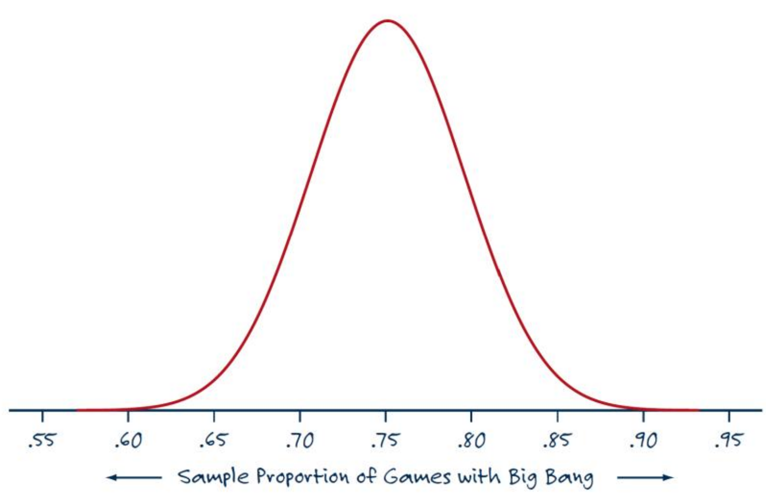
The alternative hypothesis is that less than three-fourths of all major-league baseball games contain a big bang. In symbols, the alternative hypothesis is $H_a: \pi < .75$.

To investigate this claim, we randomly selected one week of the 2006 major-league baseball season. Then we examined the 95 games played that week to determine which had a big bang and which did not.

c) Sketch and label the sampling distribution for the sample proportion of games containing a big bang, according to the Central Limit Theorem, assuming that the null hypothesis is true. Also check whether the conditions hold for the CLT to apply.

The CLT applies here because $95(.75) = 71.25$ is greater than 10 and $95(.25) = 23.75$ is also greater than 10. According to the CLT, the sample proportion would vary approximately normally with mean .75 and standard deviation

$$\sqrt{\frac{(.75)(.25)}{95}} \approx .0444$$



Of the 95 games in our sample, 47 contained a big bang.

d) Calculate the sample proportion of games in which a big bang occurred. Use an appropriate symbol to denote it.

The sample proportion of games in which a big bang occurred is

$$\hat{p} = \frac{47}{95} \approx .495$$

e) Is this sample proportion less than three-fourths and therefore consistent with Marilyn's (alternative) hypothesis? Shade the area under your sampling distribution curve corresponding to this sample result in the direction conjectured by Marilyn.

Yes, this sample proportion is less than .75, as Marilyn conjectured.

f) Calculate the test statistic and find its *P*-value.

The test statistic is

$$z = \frac{.495 - .75}{\sqrt{\frac{(.75)(.25)}{95}}} \approx \frac{.495 - .75}{.0444} \approx -5.74$$

This statistic says that the observed sample result is almost six standard deviations below what the grandfather conjectured. This z-score is way off the chart in Table II, indicating that the p-value is virtually zero.

g) Generate your conclusion at the α = 0.01 level (all three parts).

Yes, this very small p-value indicates that the sample data provide extremely strong evidence against the grandfather's claim. There is extremely strong evidence that less than 75% of all major-league baseball games contain a big bang. The null (grandfather's) hypothesis would be rejected at the $\alpha = .01$ level.

In her response, Marilyn went on to conjecture that the actual proportion of big bang games is one-half.

h) Using a two-sided alternative, state the null and alternative hypotheses (in symbols and in words) for testing Marilyn's claim.

The hypotheses for testing Marilyn's claim are $H_0: \pi = .5$ vs. $H_a: \pi \neq .5$.

i) Calculate the test statistic and use technology to determine the *P*-value for this test.

The test statistic is

$$z = \frac{.495 - .5}{\sqrt{\frac{(.5)(.5)}{95}}} \approx \frac{.495 - .5}{.0513} \approx -0.10$$

The p-value is $2(.4602) = .9204$.

j) What conclusion would you draw concerning Marilyn's conjecture (all three parts)?

This p-value is not small at all, suggesting that the sample data are quite consistent with Marilyn's hypothesis that half of all games contain a big bang. The sample data provide no reason to doubt Marilyn's hypothesis.