Introduction to Statistics
Activity 9.4c **SOLUTIONS**

Kissing Couples

Recall the study of 124 kissing couples that found 80 leaning to the right. Assume you find the following 90% and 99% confidence intervals for the population proportion of couples who lean to the right (call this population proportion p):

90% CI for p: (0.574, 0.716) and 99% CI for p: (0.534, 0.756).

a) Is the value 0.5 inside either interval? What about the value 2/3 (0.667)?
0.5: **No, neither** 0.667: **Yes, both**

Assume you do a two-sided test of whether the population proportion differs from two-thirds (0.667), and find:

H_0: p = 0.667, H_1: p ≠ 0.667, $z = 0.52$, P-value = 0.606.

b) Based on this P-value, would this test reject or fail to reject the value 0.667 at the $\alpha = 0.1$ level? What about the $\alpha = 0.01$ level?
0.10 level: **fail to reject** 0.01 level: **fail to reject**

In class, we conducted a one-sided test of whether the population proportion is greater than 0.5, but if we had conducted a two-sided test you would have found:

H_0: p = 0.500, H_1: p ≠ 0.500, $z = 3.23$, P-value = 0.001238

Note that the test statistic (z = 3.23) is the same but this P-value is actually twice the size of the one-sided P-value that we found.

c) Based on this two-sided P-value, would this test reject or fail to reject the value 0.500 at the $\alpha = 0.1$ level? What about the $\alpha = 0.01$ level?
0.10 level: **reject** 0.01 level: **reject**

d) Summarize your answers to parts a–c in the first two lines of the following table:

Hypothesized Value	Contained in 90% CI?	Contained in 99% CI?	Test Statistic	P-value	Reject at 0.10 Level?	Reject at 0.01 Level?
0.500	**a. No**	**a. No**	**3.23**	**0.00124**	**c. reject**	**c. reject**
0.667	**a. Yes**	**a. Yes**	**0.52**	**0.606**	**b. fail to reject**	**b. fail to reject**
0.725	**h. No**	**h. Yes**	**f. -1.990**	**0.04659**	**g. reject**	**g. fail to reject**

e) What do you notice about the relationship between whether a hypothesized value is in a confidence interval and whether it is rejected?

If it's in the interval, we fail to reject. If it's not in the interval, we reject.

f) Use technology to calculate the test statistic and *P*-value for a two-sided test of whether the population proportion differs from 0.725.

Test statistic: $se=\sqrt{\frac{0.725(1-0.725)}{124}} = \mathbf{0.04010}$, $z = \frac{0.6452-0.725}{0.04010} = \mathbf{-1.990}$, ***P*-value: 0.04659.**

g) Would you reject the hypothesis that the population proportion who lean to the right is 0.725 at the 0.10 level? What about the 0.01 level?

0.10 level: **Yes. Reject H_0. Conclude H_1.** 0.01 level: **No. Fail to Reject H_0.**

h) Fill in the final row of the table in part d. Elaborate on what you answered in part e, about the relationship between whether a hypothesized value is in a confidence interval and whether it is rejected. How does your test decision for a particular level of α relate to the confidence level?

This activity reveals a duality between confidence intervals for estimating a population parameter and a two-sided test of significance regarding the value of that parameter. Roughly speaking, if a 99% confidence interval for a parameter does not include a particular value, then a two-sided test of whether or not the parameter equals that particular value will be statistically significant at the α = 0.01 level. The same is true for a 90% confidence interval with the 0.10 significance level and for a 95% confidence interval with the 0.05 significance level, and so on.

Confidence intervals and hypothesis tests are complementary procedures. Whereas hypothesis tests can establish strong evidence that a parameter differs from a hypothesized value, confidence intervals estimate the magnitude of that difference.