Introduction to Statistics
Activity 8.3b **SOLUTIONS:**

Percent of Folks w/Cellphones

1. Suppose a market research firm is hired to estimate the percentage of adults living in a large city who have cell phones. Five hundred adult residents are chosen in a simple random sample from the city and are surveyed to determine whether they have cell phones. Of the 500 people sampled, 421 responded yes - they own cell phones, the rest say they don't. Using a 95% confidence level, compute a confidence interval for the true proportion of adult residents of this city who have cell phones.

z*–values for Various Confidence Levels

Confidence Level	z*-value
80%	1.28
90%	1.645 (by convention)
95%	1.96
98%	2.33
99%	2.58

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

   1. What are the sample size and sample proportion? Use the correct notation.
      n=500 $\hat{p} = \frac{421}{500} = 0.842$
   2. What's the estimated standard error of the sample proportion?
      $\widehat{se} = \sqrt{\frac{0.842(1-0.842)}{500}} = 0.01631$
   3. What is the critical value (z^*)? **z^*=1.96**
   4. What is the margin of error?
      moe = $z^*(\widehat{se})$ = 1.96(0.01631) = 0.03197
   5. What is the confidence interval?
      $\hat{p} \pm \text{moe} = 0.842 \pm 0.03197 = (0.8100, 0.8740)$
   6. Describe this CI in context (provide a sentence, it should mention cell phones!).
      We are 95 percent confident that the percentage of adult residents in the city that have cell phones is between 0.810 and 0.874.
   7. What do we mean by 95% confident?
      If we were to do many such samplings of the residents, and generate confidence intervals for each sample, 95 percent of those confidence intervals would contain the population parameter (the true proportion of residents who own cell phones).

2. Another 500 people are sampled. This time, 460 of the respondents said that they owned a cell phone.
   1. What is the sample proportion?
      $\hat{p} = \frac{460}{500} = 0.92$

2. Was this result likely to happen according to our previous result?
   No.

3. What do you think about your confidence interval?
   It wasn't accurate.

3. The organizers want to settle this once and for all. They want a survey done that will have only 1% margin of error with a 99% confidence level! Below, you'll calculate the necessary sample size.
   1. Calculate the average of the two sample proportions. Use this as your new sample proportion.

$$\hat{p} = \frac{0.842 + 0.92}{2} = 0.881$$

   2. Write down some potentially useful math expressions related to this question's introduction.
   $z^*(\widehat{se}) = \text{moe} = 0.01,$ $z^* = 2.58.$

   So, $0.01 = z^*(\widehat{se}) = 2.58\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

   1. From these, fill in constants you know, and attempt to solve for the unknown sample size n.
   So $0.01 = 2.58\sqrt{\frac{0.881(1-0.881)}{n}} = 2.58\sqrt{\frac{0.104839}{n}}$

$$\left(\frac{0.01}{2.58}\right)^2 = \frac{0.104839}{n}$$

$$n\left(\frac{0.01}{2.58}\right)^2 = 0.104839$$

$$n = \frac{0.104839}{\left(\frac{0.01}{2.58}\right)^2} \approx 6979$$

   4. Report your result in context.
   To get 99 percent confidence with a one percent margin of error, we would need a sample size of 6979 residents!

   5. The city's population is 450,000. Can we meet the technical requirements for CLT?
   To employ CLT, we need $n\hat{p} \geq 10$ *and* $n(1-\hat{p}) \geq 10$ **. Observe that** $6979\,(0.881) = 6148 \geq 10,$ *and* $n(1-\hat{p}) = 831 \geq 10.$ **No problem except for the expense in doing such a sample.**