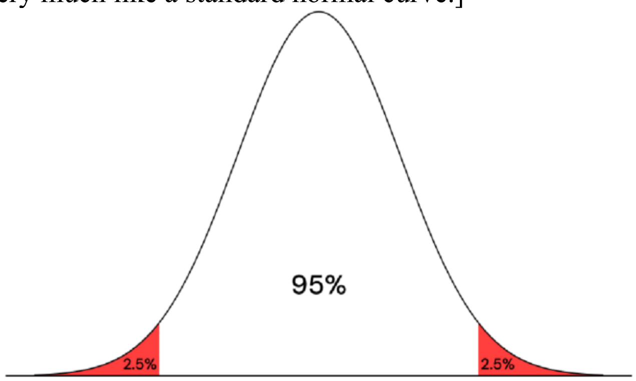
Introduction to Statistics
Activity 8.2a **SOLUTIONS**

Critical Values for Means

a) Draw a rough sketch of the t-distribution with 9 degrees of freedom.
[Hint: It should look very much like a standard normal curve.]



b) The critical value t* for a 95% confidence interval is the value such that 95% of the area under the curve is between
t* and -t*. Shade this area on your sketch.

c) What is the area to the right of t* under the curve? [Hint: This area is not 0.05.]
0.025

d) Look at the t-dist Calculator (Statdistributions.com/t/) (w/two tails and p-value being the shaded area under the curve) to find the value of t* with that area to its right under a t-distribution with 9 degrees of freedom. Report this value.
$t_9 = 2.263$

e) Is this critical value less than or greater than the critical value z* from the standard normal distribution for a 95% confidence interval? Explain why this is helpful, based on your motivation for needing the
t-distribution instead of the z-distribution.

95%: $z^* = 1.96 < 2.262 = t_9$. This is helpful since the uncertainty from using the distribution of the sample, instead of the population, results in us needing a wider interval to ensure we capture the population mean 95% of the time.

t* greater than counterparts from the z^*, reflecting greater uncertainty estimating σ with s.

Makes CIs just long enough that 95% capture value of the population mean.

f) Find the critical value t* for a 95% confidence interval based on a sample size of n=20 (from the calculator). How does this value compare to the previous t* value? Explain why this is appropriate for the interval procedure as well.
[Hint: Think about whether a larger sample size would increase/decrease uncertainty in estimating σ by s.]
95%: $t_{19} = 2.093 < 2.262 = t_9$. Having a larger sample size decreases the uncertainty in the dist, and therefore should be associated with a smaller interval size, which this new t-value provides.

g) Find the critical value t* for a 90% and 99% confidence interval, based on a sample size of n=20. Which is greater? Explain why this is appropriate.

90%: $t_{19} = 1.729$.   99%: $t_{19} = 2.861$.

99% is greater, this is appropriate since we expect it to be more likely to capture the population mean if the interval is bigger.

h) Find the critical value t* for a 95% confidence interval based on a sample size of n=130.
95%: $t_{129} = 1.979$

i) How has the t* value changed as you increased the degrees of freedom by increasing the sample size? How does t* with 129 degrees of freedom compare to z*?
The t-value decreases as you increase the sample size. $Z < t_{129} < t_{19} < t_9$.

When n is large, there's a small penalty for σ in s, and so t* converges to z*. Looking up z* (calculator.net): z* = 1.96.