Introduction to Statistics I

Textbook: Elementary Statistics (4th Edition, by Navidi and Monk), and Workshop Statistics (4th Edition, by Rossman and Chance).

Previous Lecture

- Six steps of significance test for qualitative vars (proportions)

§9.7: Experimental Power

Recall: Six Steps of Significance Testing

1. Lay out problem (param, var, & pop) and check technical conditions.
2. State hypotheses H_0, H_1
3. Set Sig-level α
4. Calculate test statistic (t/z-score)
5. Calculate P-value
6. State Conclusion: evidence, decision, in-context summary.

Errors

When we reject, or fail to reject the null, we do so based on a sig-level α.
But this leaves room for error, so we'll never *know* if we're right!

What does it mean to be right?

In Reality: The null must either be true or false.

If the null is true, what's the correct decision?

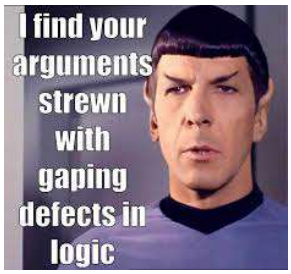
If the null is false, what's the correct decision?

	Statistical Test Decision	
	Fail to reject null	Reject null
If null is true	Correct	
If null is false		Correct

We commit **Type I Error** (**T1E**) when we reject a true null.

We commit **Type II Error** (**T2E**) when we fail to reject a false null.

	Statistical Test Decision	
	Fail to reject null	Reject null
If Null is true	Correct	T1E
If Null is false	T2E	Correct



Previous Chocolate Pen Example: When $\alpha = 0.05$, we rejected the null and concluded that more that 45% of new businesses in Syracuse are still in business after 5 years. Of the two errors, which of them *could* we have made?

If we made an error, it had to be T1E.

Such an error happens due to the randomness of our sampling. An unsual sample can cause us to conclude the proportion is greater than 0.45 when it is actually equal to 0.45.

Type 1 Errors

We make a T1E when we reject the null even though the null is true.

Fact: The sig. level α, is equal to probability of committing a T1E.

So, by keeping α small, we can prevent T1Es!

Example: Suppose in our previous example that with $\alpha = 0.01$ we *fail to reject* the null and thus can't conclude that more that 45% of new businesses in Syracuse are still in business after 5 years.

Of the two errors, which of them *could* we have made?

If we made an error here, it is T2E.

In this case, the true proportion *is* greater than 0.45, but we were unable to determine that based on our data.

Type 2 Error

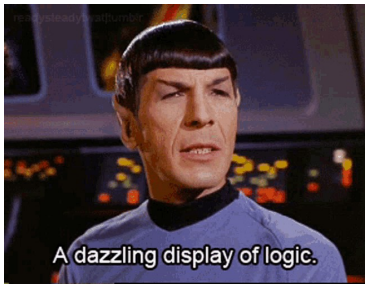
T2E is when we fail to reject a false null.

Note: smaller sig. levels (α) require smaller P-values to reject a false null;
and thus have a higher chance of producing a T2E.

So, when we reduce the sig. level (α) to prevent T1E, we *increase* the likelihood of T2E!

However, we can reduce the probability of a T2E by *gathering more evidence*.
Higher sample sizes lower the likelihood of T2E.

So, set a small α to avoid a T1E, then collect as large a sample as possible to prevent a T2E.



Power

Before beginning a study, we want to know the probability that we'll identify a false null (assuming it's false).

This probability is called the **power** of a study, and is 1 minus the T2E probability:
$power = 1 - T2E$.

This is important to compute *before* conducting the study, because there's no point in doing the study if our sample size is so small that we can't determine if the null is false.

Looking at the expression ($power = 1 - T2E$), notice that we **increase power** by **reducing T2E** rate.
And we reduce T2E by **increasing the sample size**.

A power of $0.8 - 0.9$ is considered acceptably large in most situations.

	Statistical Test Decision	
	Fail to reject null	Reject null
If Null is true	Correct	T1E (prob is α)
If Null is false	T2E (prob depends on n)	Correct (prob/power: 1-T2E)

"Logic is the beginning of wisdom, not the end"

Activity 9.7a

Activity: 9.4c

What did we learn?

- Errors T1E/T2E
- How to minimize errors
- Power