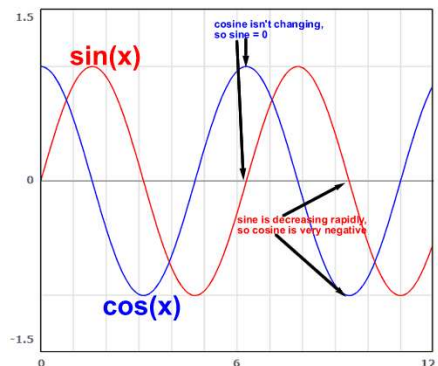


Calculus I

Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

3.3 - Derivatives of Trigonometric Functions



Recall that $\csc = \frac{1}{\sin}$, $\sec = \frac{1}{\cos}$, and $\cot = \frac{1}{\tan} = \frac{\cos}{\sin}$.

Using the limit definition of a derivative, it is known that:

$$(\sin x)'|_{x=0} = \lim_{x \rightarrow 0} \frac{\sin(x-0) - \sin 0}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$$

$$\text{and } (\cos x)'|_{x=0} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0.$$

So this must mean that at $x = 0$, we have $(\sin x)' = 1 = \cos x$, and $(\cos x)' = 0 = -\sin x$.

More generally for all x we have:

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$

Problem 3. Differentiate $f(x) = \sin x + \frac{1}{2} \cot x$.

$$f(x) = \sin x + \frac{1}{2} \frac{\cos x}{\sin x}$$

$$f' = \cos x + \frac{1}{2} \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$f' = \cos x + \frac{1}{2} \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

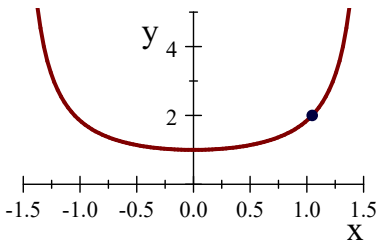
$$f' = \cos x - \frac{1}{2} \frac{1}{\sin^2 x}$$

$$f' = \cos x - \frac{1}{2} \csc^2 x \quad [\text{ or just memorize the part of the review for } (\cot x)']$$

Problem 9. Differentiate $y = \frac{x}{2-\tan x}$.

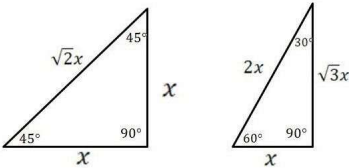
$$\begin{aligned}
 y' &= \frac{(1)(2-\tan x)-x(-\sec^2 x)}{(2-\tan x)^2} \\
 &= \frac{2-\tan x+x\sec^2 x}{(2-\tan x)^2}.
 \end{aligned}$$

Problem 21. Find an equation of the tangent line to $y = \sec x$ at the point $(\frac{\pi}{3}, 2)$



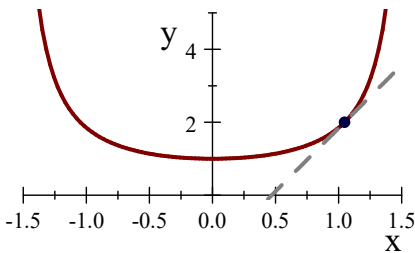
$$y' = \sec x \tan x.$$

$$\begin{aligned}
 \text{So } y' \Big(\frac{\pi}{3} \Big) &= \sec \frac{\pi}{3} \tan \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} \\
 &= \frac{1}{\frac{1}{2}} \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}.
 \end{aligned}$$



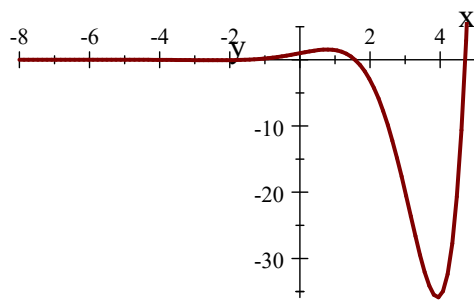
Equation of the tangent line:

$$y - 2 = 2\sqrt{3} \Big(x - \frac{\pi}{3} \Big) \quad \text{or} \quad y = 2\sqrt{3} x + 2 - \frac{2\pi}{3} \sqrt{3}.$$



$$\sec x \text{ and } 2\sqrt{3} x + 2 - \frac{2\sqrt{3} \pi}{3}$$

Problem 34. For what values of x does the graph of $f = e^x \cos x$ have a horizontal tangent?



$f(x) = e^x \cos x$ has a horizontal tangent when $f' = 0$.

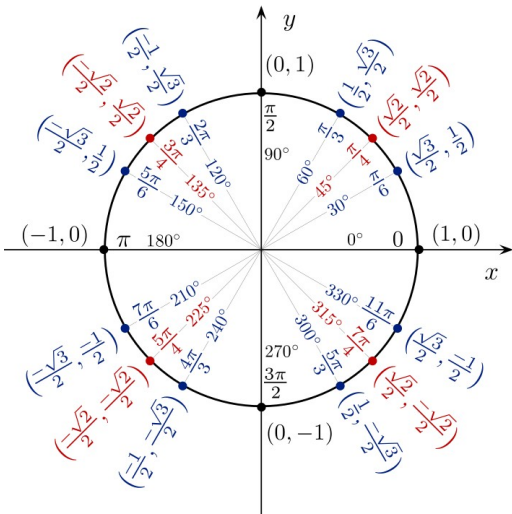
$$f' = e^x(\cos x) + e^x(-\sin x)$$

$$= e^x(\cos x - \sin x).$$

$$f' = 0 \Rightarrow \cos x - \sin x = 0$$

$$\Rightarrow \cos x = \sin x$$

$$\Rightarrow x = \frac{\pi}{4} + n\pi, \text{ where } n \text{ an integer.}$$



Problem 43. Find $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x}$.

Since we already know that $\lim_{3x \rightarrow 0} \frac{\sin 3x}{3x} = 1$, we would prefer to solve something in the form: $\lim_{3x \rightarrow 0} \left[\frac{\sin 3x}{3x} \cdot g(x) \right]$, for some mathematical expression $g(x)$.

So we want $\frac{\sin 3x}{5x^3 - 4x} = \frac{\sin 3x}{3x} \cdot g(x)$.

Solving for $g(x)$, we find: $g(x) = \frac{3x \sin 3x}{5x^3 - 4x} = \frac{3}{5x^2 - 4}$.

So, $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x} =$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \cdot \frac{3}{5x^2 - 4} \right)$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{3}{5x^2 - 4}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{3}{5x^2 - 4}$$

Where in the first limit, we did a change of variable $x \rightarrow \frac{1}{3}x$.

$$= 1 \cdot \left(\frac{3}{-4} \right) = -\frac{3}{4}.$$

