

Calculus I

Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

3.1 - Derivatives of Polynomials and Exponential Functions

Derivatives of...

♦ **Constant Functions:** If $y = c$, where c is some constant, then $y' = 0$.

♦ **Power Functions:** If $f(x) = x^r$, where r is a real number, then $\frac{d}{dx}(x^r) = rx^{r-1}$.

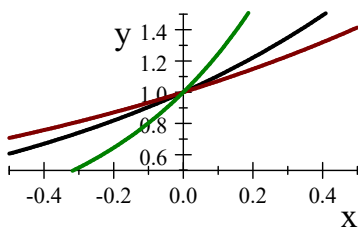
This includes if $f(x) = x$, where $\frac{d}{dx}(x^1) = 1 \cdot x^0 = 1$.

♦ **Constant Multiple:** If c is a constant and f is a differentiable function,

then $\frac{d}{dx}[cf(x)] = c \frac{d}{dx}f(x)$.

♦ **Sum/Difference:** If f and g are both differentiable,

then $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$.



2^x , 2.718^x , and 3^x

♦ **Euler's Number e** : Imagine the family of functions: $f(x) = a^x$, where a is any real number.

e is defined to be the number such that for $f(x) = a^x$, we have:

$$f'(0) = \lim_{h \rightarrow 0} \frac{a^{0+h} - a^0}{h} = \lim_{h \rightarrow 0} \frac{a^h - 1}{h} = 1. \text{ (The slope is 1 at } x = 0. \text{ The number turns out to be } e \approx 2.718).$$

♦ **Exponential Functions:** If $f(x) = e^x$, then $f'(x) = e^x$. More generally, if $f(x) = a^x$, then $f'(x) = a^x \ln(a)$.

♦ **Polynomials:** Using Constant Multiple Rule and Sum/Difference Rule

Say $y = ax^2 + bx^3 - c$, where a, b, c are some constants. Then:

$$\begin{aligned} y' &= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx^3) - \frac{d}{dx}(c) \\ &= a \frac{d}{dx}(x^2) + b \frac{d}{dx}(x^3) - 0 \\ &= 2ax + 3bx^2. \end{aligned}$$

Random Problem. Find the derivative of: $f(x) = 7x^2 - 7x^3 + 7e^x$

$$\frac{d}{dx}f(x) = \frac{d}{dx}[7x^2 - 7x^3 + 7e^x]$$

$$= 7 \frac{d}{dx} [x^2 - x^3 + e^x]$$

$$= 7 \left[\frac{d}{dx} (x^2) - \frac{d}{dx} (x^3) + \frac{d}{dx} (e^x) \right]$$

$$= 7[2x - 3x^2 + e^x]$$

$$= 14x - 21x^2 + 7e^x$$

Random Problem. Find the derivative of: $f(t) = 1.7t^{2.3} - \frac{4}{\sqrt[3]{t}}$

$$f' = \frac{d}{dt}(1.7t^{2.3}) - \frac{d}{dt}\left(\frac{4}{\sqrt[3]{t}}\right)$$

$$f' = 1.7 \frac{d}{dt}(t^{2.3}) - 4 \frac{d}{dt}\left(t^{-\frac{1}{3}}\right)$$

$$f' = 1.7(2.3)t^{1.3} - 4\left(-\frac{1}{3}\right)t^{-\frac{4}{3}}$$

$$f' = 3.91t^{1.3} + \frac{4}{3}t^{-\frac{4}{3}}.$$

Second Derivative...

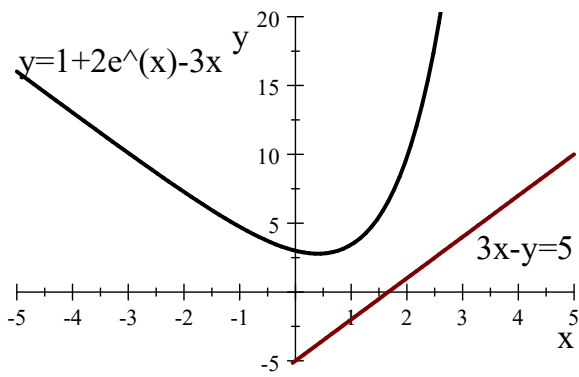
$$f'' = (f')' = \left(3.91t^{1.3} + \frac{4}{3}t^{-\frac{4}{3}}\right)' = (3.91t^{1.3})' + \left(\frac{4}{3}t^{-\frac{4}{3}}\right)'$$

$$f'' = 3.91(t^{1.3})' + \frac{4}{3}\left(t^{-\frac{4}{3}}\right)'$$

$$f'' = 3.91(1.3)t^{0.3} + \frac{4}{3}\left(-\frac{4}{3}\right)t^{-\frac{7}{3}}$$

$$f'' = 5.083t^{0.3} - \frac{16}{9}t^{-\frac{7}{3}}.$$

Random Problem. At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?



$$-y = 5 - 3x$$

$$y = 3x - 5$$

Slope is 3

$$y' = \frac{d}{dx}(1) + 2 \frac{d}{dx}(e^x) - 3 \frac{d}{dx}(x)$$

$$y' = 0 + 2e^x - 3(1) = 2e^x - 3$$

$$2e^x - 3 = 3, \quad 2e^x = 6, \quad e^x = 3$$

So, $\ln(e^x) = \ln 3$. Recall that: $\ln(a^x) = x \ln a$, for all $a > 0$.

Therefore: $\ln(e^x) = x = \ln 3 \approx 1.1$.

And this is the x value where the tangent line to the curve is parallel to the line.

Part b.) Find the tangent line, and Illustrate.

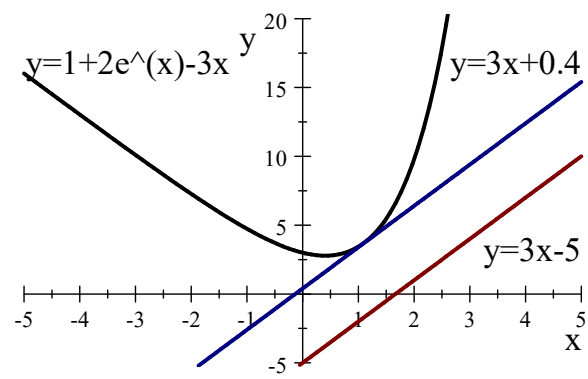
We know the slope of the tangent line, and an x -value, and now we just need a y -value in order to complete the point slope form.

$$y = 1 + 2e^{\ln 3} - 3(\ln 3) = 1 + 2 \cdot 3 - 3(\ln 3) \approx 3.7.$$

So our point is: $(x, y) \approx (1.1, 3.7)$, and our line equation is:

$$y - 3.7 = 3(x - 1.1) = 3x - 3.3, \text{ or}$$

$$y = 3x + 0.4.$$



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Materials for Other Courses Found at **MathTalker.org**