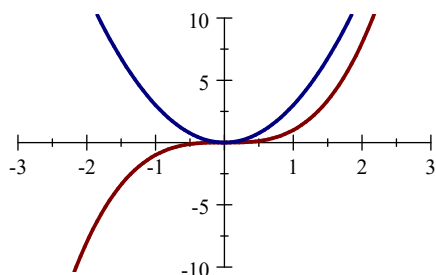


Calculus I

Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

2.8 - Derivative as a Function

f' as a Function: Given any point x at which the derivative $f'(x)$ exists, we assign to x the number $f'(x)$ (the slope of f at x). In this way, we regard f' as a function in-and-of itself, and we call the function f' , **the derivative of f** .

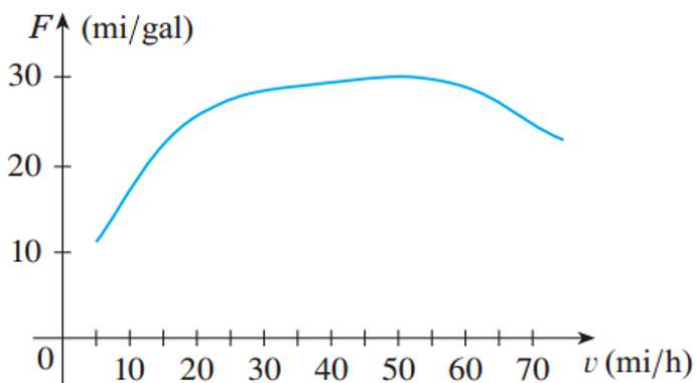


$$f = x^3 \text{ and } f' = 3x^2$$

Definition: A function f is **differentiable at a** if $f'(a)$ exists. We say that f is **differentiable on an open interval (a, b)** [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every point in the interval.

Differentiability Implies Continuity: If f is differentiable at a , then f is continuous at a .

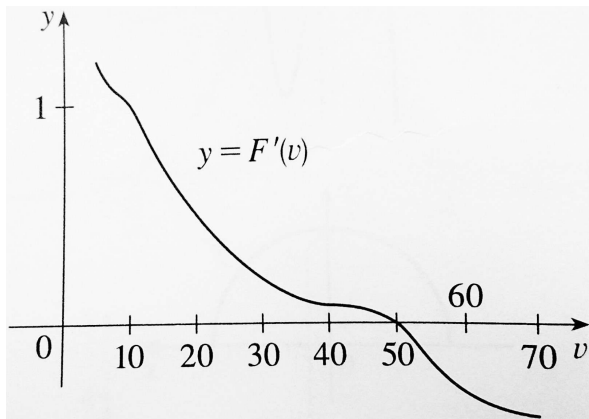
Problem 14. The graph below (from the US Department of Energy) shows how driving speed affects gas mileage. Fuel economy F is measured in miles per gallon and speed v is measured in miles per hour.



a. What is the meaning of the derivative $F'(v)$?

$F'(v)$ is the instantaneous rate of change of fuel economy with respect to speed.

b. Sketch the graph of $F'(v)$.



c. At what speed should you drive if you want to save on gas?

Problem 21. Find the derivative of $f(x) = \frac{1}{2}x - \frac{1}{3}$ using the definition of derivative.

State the domain of the function and the domain of its derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left[\frac{1}{2}(x+h) - \frac{1}{3}\right] - \left(\frac{1}{2}x - \frac{1}{3}\right)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}h}{h} = \lim_{h \rightarrow 0} \frac{1}{2} = \frac{1}{2}. \end{aligned}$$

Domain of function $\frac{1}{2}x - \frac{1}{3}$?

Domain of derivative?

$$f = \frac{1}{2}x - \frac{1}{3}, \quad f' = \frac{1}{2}$$

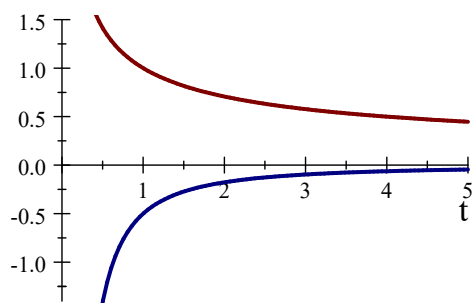
Problem 26. Find the derivative of $g(t) = \frac{1}{\sqrt{t}}$ using the definition of derivative.

State the domain of the function and the domain of its derivative.

$$\begin{aligned}
 f' &= \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} \cdot \frac{\sqrt{t} \sqrt{t+h}}{\sqrt{t} \sqrt{t+h}} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{t} - \sqrt{t+h}}{h \sqrt{t} \sqrt{t+h}} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{t} - \sqrt{t+h}}{h \sqrt{t} \sqrt{t+h}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} = \lim_{h \rightarrow 0} \frac{t - (t+h)}{h \sqrt{t} \sqrt{t+h} (\sqrt{t} + \sqrt{t+h})} = \lim_{h \rightarrow 0} \frac{-h}{h \sqrt{t} \sqrt{t+h} (\sqrt{t} + \sqrt{t+h})} \quad (\text{did you notice the difference of squares?}) \\
 &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{t} \sqrt{t+h} (\sqrt{t} + \sqrt{t+h})} = \frac{-1}{\sqrt{t} \sqrt{t} (\sqrt{t} + \sqrt{t})} = \frac{-1}{2t\sqrt{t}}
 \end{aligned}$$

Domain of function $\frac{1}{\sqrt{t}}$?

Domain of derivative?



$$f = \frac{1}{\sqrt{t}}, \quad f' = \frac{-1}{2t\sqrt{t}}$$

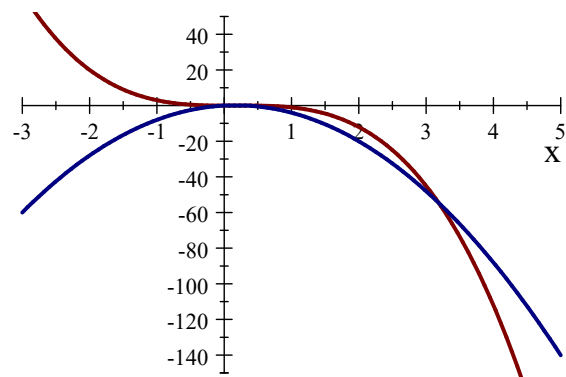
Problem 25. Find the derivative of $f(x) = x^2 - 2x^3$ using the definition of derivative.

State the domain of the function and the domain of its derivative.

$$\begin{aligned}
 f' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 2(x+h)^3] - (x^2 - 2x^3)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2) + (-2x^3 - 6x^2h - 6xh^2 - 2h^3) - (x^2 - 2x^3)}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - 6x^2 - 6xh - 2h^2)}{h} = \lim_{h \rightarrow 0} (2x + h - 6x^2 - 6xh - 2h^2) = 2x - 6x^2.
 \end{aligned}$$

Domain of function $x^2 - 2x^3$?

Domain of derivative?



$$f = x^2 - 2x^3, \quad f' = 2x - 6x^2$$

Prepared by Dr. Jodin Morey.

Materials for Other Courses Found at **MathTalker.org**