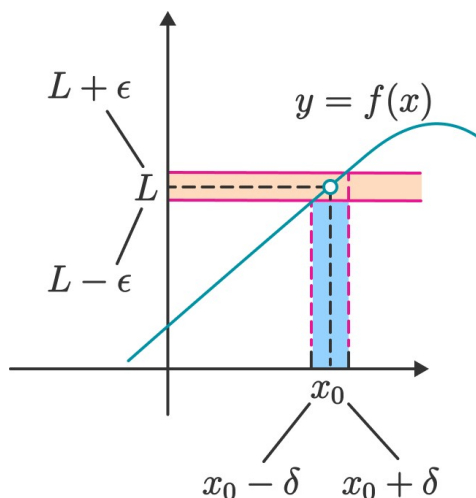


Calculus I

Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

2.4 - Precise Definition of a Limit



Limit: We say that $\lim_{x \rightarrow a} f(x) = L$ if, for every number $\epsilon > 0$, there exists a number $\delta > 0$ such that; if $(0 < |x - a| < \delta)$ then $(|f(x) - L| < \epsilon)$.

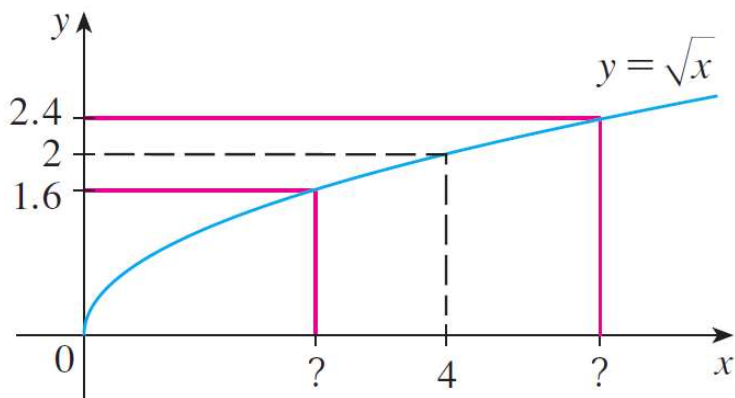
Left-Handed Limit: We say that $\lim_{x \rightarrow a^-} f(x) = L$ if, for every number $\epsilon > 0$, there exists a number $\delta > 0$ such that; if $(a - \delta < x < a)$, then $(|f(x) - L| < \epsilon)$.

Right-Handed Limit: We say that $\lim_{x \rightarrow a^+} f(x) = L$ if, for every number $\epsilon > 0$, there exists a number $\delta > 0$ such that; if $(a < x < a + \delta)$ then $(|f(x) - L| < \epsilon)$.

Infinite Limits: We say that $\lim_{x \rightarrow a} f(x) = \infty$ if, for every **positive** number M , there exists a **positive** number $\delta > 0$ such that; if $(0 < |x - a| < \delta)$ then $(f(x) > M)$.

Similarly, We say that $\lim_{x \rightarrow a} f(x) = -\infty$ if, for every **negative** number N , there exists a **positive** number $\delta > 0$ such that; if $(0 < |x - a| < \delta)$ then $(f(x) < N)$.

Problem 3. Use the given graph of $f(x) = \sqrt{x}$ to find a number δ such that if $|x - 4| < \delta$ then $|\sqrt{x} - 2| < 0.4$



$$1.6^2 = 2.56$$

$$2.4^2 = 5.76$$

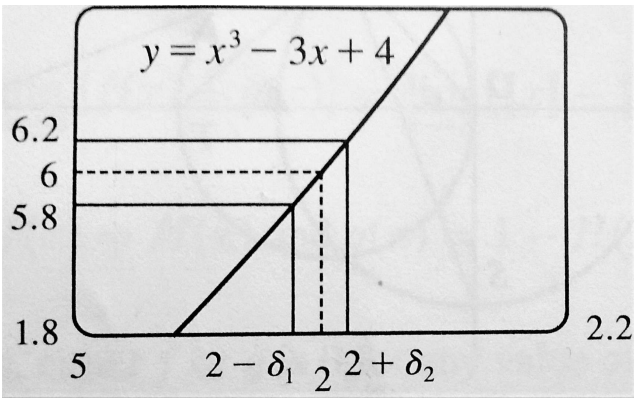
$$|x - 4| < |2.56 - 4| = 1.44$$

$$|x - 4| < |5.76 - 4| = 1.76$$

$$\delta < 1.44$$

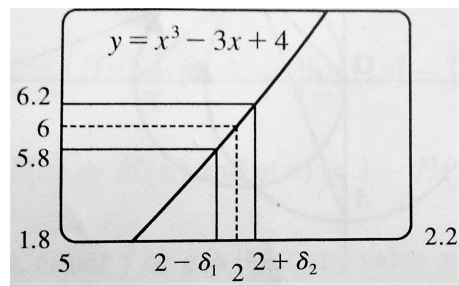
Problem 7. For the limit, $\lim_{x \rightarrow 2} (x^3 - 3x + 4) = 6$, illustrate the following definition, by finding values of δ that correspond to $\varepsilon = 0.2$.

Definition 2 (from the book): $\lim_{x \rightarrow 2} f(x) = 6$ if, for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that, if $(0 < |x - 2| < \delta)$ then $(|f(x) - 6| < \varepsilon)$.



For what x-value does the function output 5.8?
 $y = x^3 - 3x + 4 = 5.8$ when $x \approx 1.9774$,

$$\text{so } 2 - \delta_1 \approx 1.9774 \Rightarrow \delta_1 \approx 0.0226.$$



Also, $y = x^3 - 3x + 4 = 6.2$ when $x \approx 2.022$, so $2 + \delta_2 \approx 2.0219 \Rightarrow \delta_2 \approx 0.0219$.

Choosing the smaller of the two numbers, we get $\delta = 0.0219$.