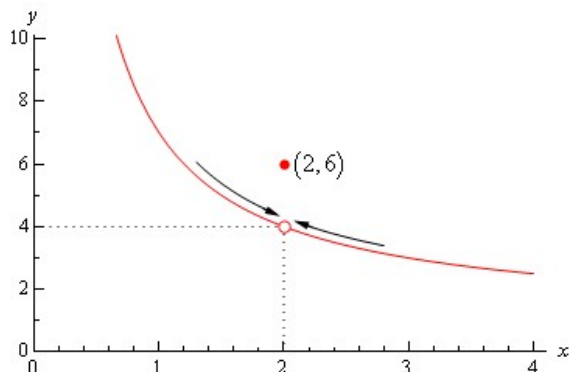


Calculus I

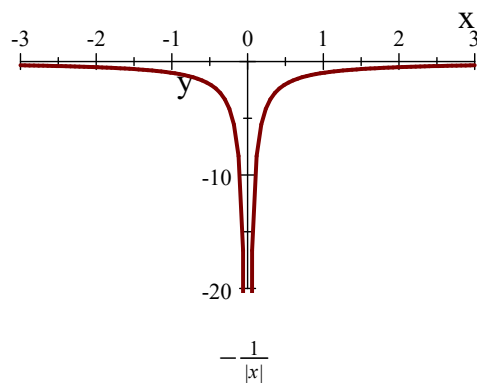
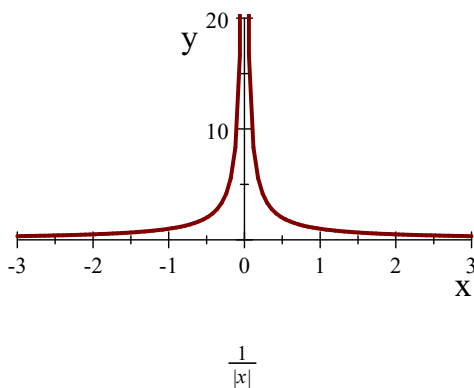
Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

2.2 - Limit of a Function

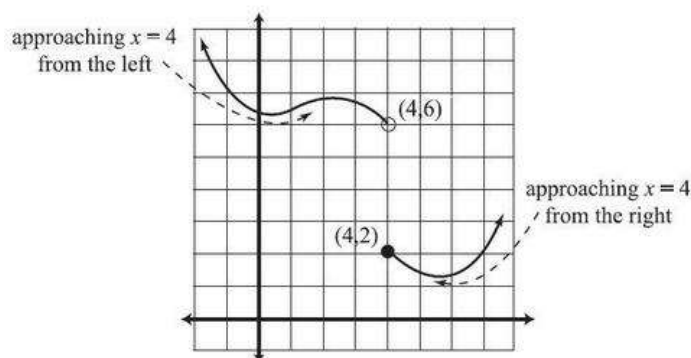


Limit: Suppose $f(x)$ is defined when x is **near** the number a . This means that f is defined on some open interval (x_0, x_1) that contains a , except possibly not defined at a itself. Then, if we can make the values of $f(x)$ **arbitrarily close to some number** $y = L$ (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a , then we write: $\lim_{x \rightarrow a} f(x) = L$, and say "the **limit** of $f(x)$, as x approaches a , equals L ."

Positive Infinite Limit: Let f be a function defined on both sides of a , except possibly at a itself. Then, we define $\lim_{x \rightarrow a} f(x) = \infty$ to mean that the value of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a , but not equal to a .

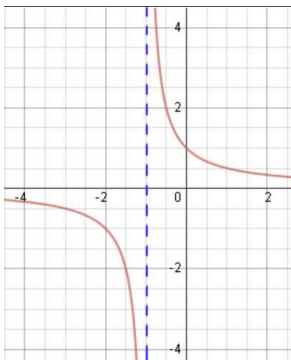


Negative Infinite Limit: Let f be defined on both sides of a , except possibly at a itself. Then, we define $\lim_{x \rightarrow a} f(x) = -\infty$ to mean that the values of $f(x)$ can be made an arbitrarily large negative number by taking x sufficiently close to a , but not equal to a .

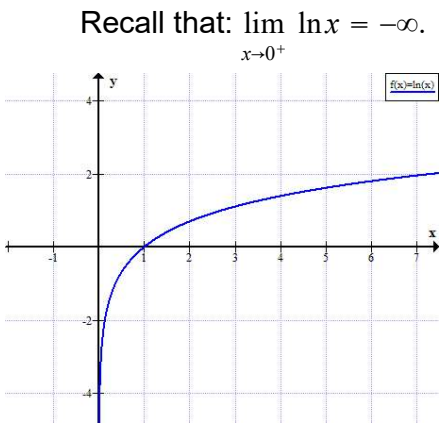


L.H.L. = 6, R.H.L. = 2

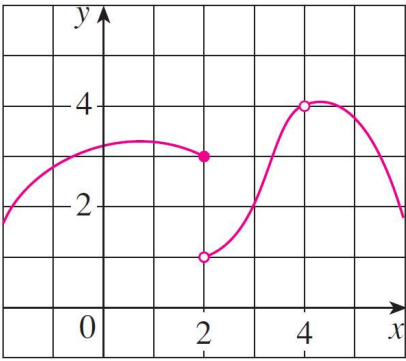
Left Hand Limit: Let f be defined on the left side of a , except possibly at a itself. Then, we define $\lim_{x \rightarrow a^-} f(x) = L$ to mean that the values of $f(x)$ can be made an arbitrarily close to L by taking x sufficiently close to a (from the left), but not equal to a . A right-hand limit $\lim_{x \rightarrow a^+}$ is defined similarly.



Vertical Asymptote: The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statements is true: $\lim_{x \rightarrow a} f(x) = \pm\infty$, $\lim_{x \rightarrow a^-} f(x) = \pm\infty$, $\lim_{x \rightarrow a^+} f(x) = \pm\infty$.



Problem 4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.



$$\lim_{x \rightarrow 2^-} f(x)$$

$$\lim_{x \rightarrow 2^+} f(x)$$

$$\lim_{x \rightarrow 2} f(x)$$

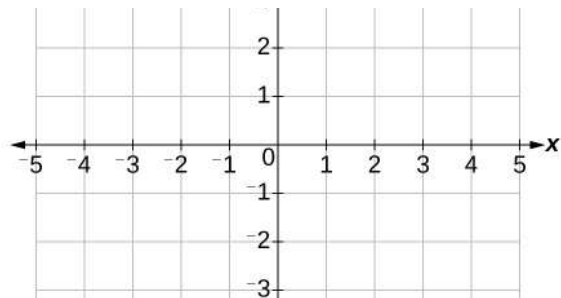
$$f(2)$$

$$\lim_{x \rightarrow 4} f(x)$$

$$f(4)$$

Problem 16. Sketch the graph of an example function f that satisfies the conditions:

$$\lim_{x \rightarrow 0} f(x) = 1, \quad \lim_{x \rightarrow 3^-} f(x) = -2, \quad \lim_{x \rightarrow 3^+} f(x) = 2, \quad f(0) = -1, \quad f(3) = 1$$



Problem 33. Determine the infinite limit: $\lim_{x \rightarrow 3^+} \ln(x^2 - 9)$.

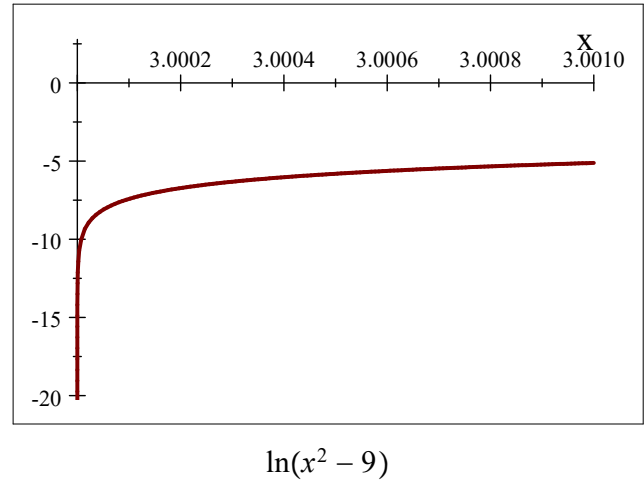
Recall we know that: $\lim_{x \rightarrow 0^+} \ln x = -\infty$.

So, let's use the substitution: $t = x^2 - 9$.

Then as $x \rightarrow 3^+$, $t \rightarrow 0^+$.

("Then as x approaches 3 from the right, t approaches 0 from the right")

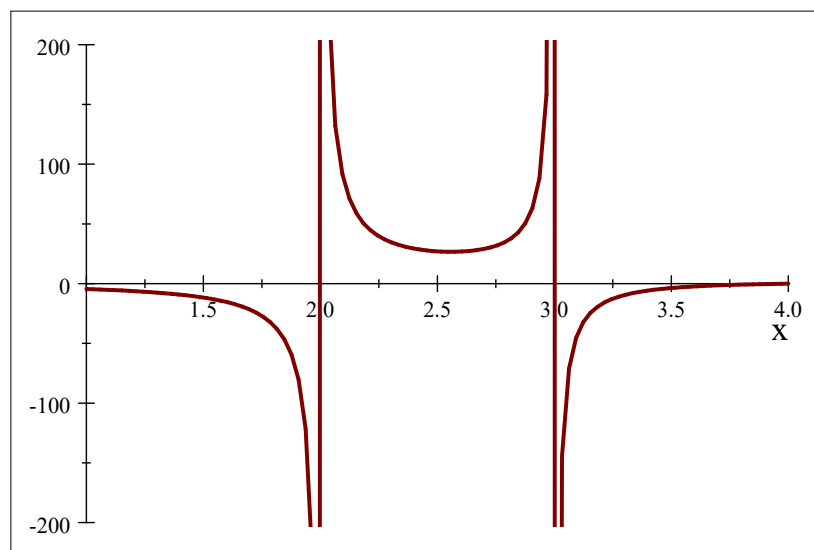
So, we can make the substitutions into $\lim_{x \rightarrow 3^+} \ln(x^2 - 9)$, to get $\lim_{t \rightarrow 0^+} \ln t = -\infty$.



Problem 37. Determine the infinite limit: $\lim_{x \rightarrow 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$

$$= \lim_{x \rightarrow 2^+} \frac{(x-4)(x+2)}{(x-3)(x-2)} \rightarrow \frac{(-2^+)(4^+)}{(-1^+)0^+} = +\infty.$$

We get the last equality since the numerator is negative, and the denominator approaches 0 through negative values and $x \rightarrow 2^+$.



$$\frac{x^2-2x-8}{x^2-5x+6}$$

Prepared by Dr. Jodin Morey.

Materials for Other Courses Found at **MathTalker.org**