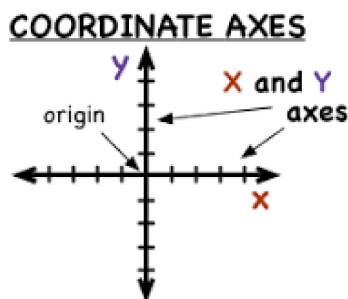


Calculus II

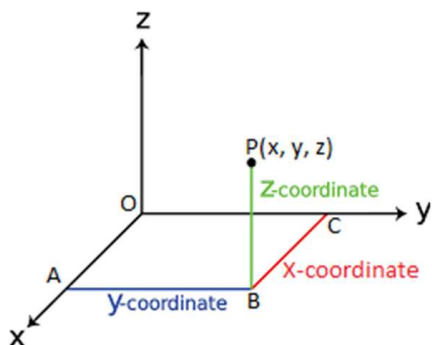
Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

12.1 - Three-Dimensional Coordinate Systems

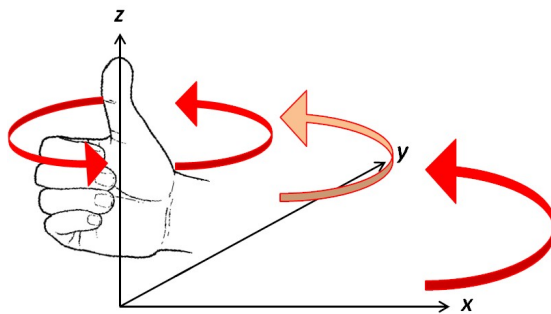


Coordinate Axes: Directed lines (for example, the x -axis, y -axis, z -axis) through the origin which are perpendicular to each other.

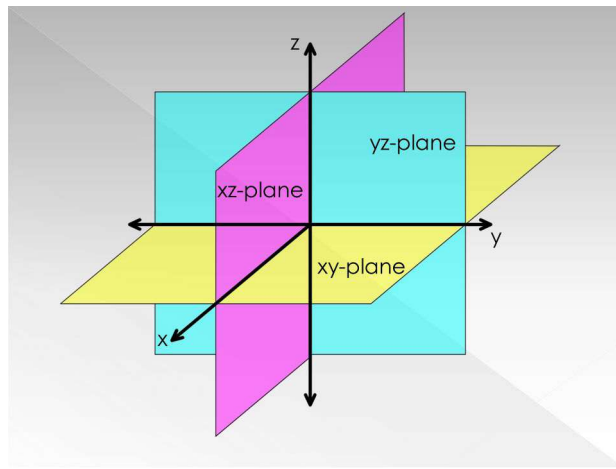
Coordinates: If we represent a point P in three-dimensional space as the **ordered triplet** (a, b, c) , where a is the distance along the x -axis, b is the distance along the y -axis, and c is the distance along the z -axis; we call a, b, c the coordinates of P .



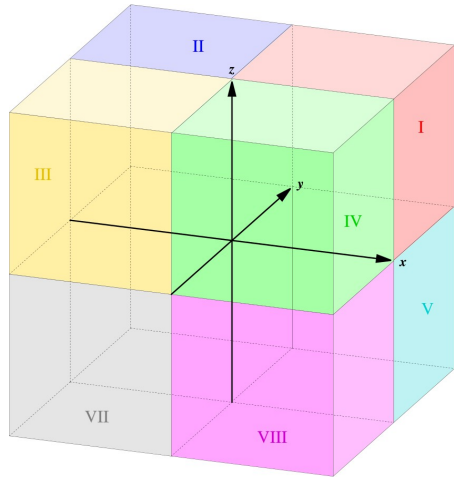
Right-Hand Rule: If you imagine curling the fingers of your right hand around the z -axis in the direction of a 90° counterclockwise rotation, starting from the positive x -axis, and continuing to the positive y -axis, then your thumb ends up pointing in the direction of the positive z -axis.



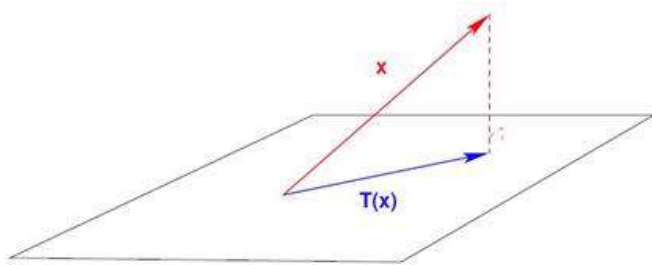
Coordinate Planes: If you take two of the axes (for example, x, y), and take all of the points either on or directly between the two axes, you end up with a coordinate plane (for example, the xy -plane).



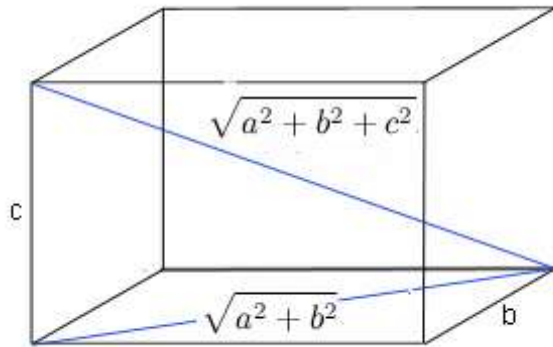
Octants: The coordinate planes divide three-dimensional space into eight areas, these areas are called octants, and the octant with positive entries for all three coordinates is called the **first octant**.



Projection: For any point (a, b, c) in three-dimensional space, we call $(a, b, 0)$ a projection of P on to the xy -plane, we call $(a, 0, c)$ a projection onto the xz -plane, and so on.



Distance Formula in Three Dimensions: The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.



Equation of a Sphere: An equation of a sphere with center $C(h, k, l)$ and radius r is $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

Problem #8 Let $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$ be the vertices of a triangle PQR . Find the lengths of the sides of the triangle. Is it a right triangle? Is it an isosceles triangle?

$$\begin{aligned} |PQ| &= \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2} \\ &= \sqrt{(4 - 2)^2 + (1 - (-1))^2 + (1 - 0)^2} = \sqrt{4 + 4 + 1} = 3. \end{aligned}$$

$$|QR| = \sqrt{(4 - 4)^2 + (1 + 5)^2 + (1 - 4)^2} = \sqrt{0 + 36 + 9} = 3\sqrt{5} \approx 6.7$$

$$|RP| = \sqrt{(4 - 2)^2 + (-5 + 1)^2 + (4 - 0)^2} = \sqrt{4 + 16 + 16} = 6.$$

Obviously not isosceles. Right triangle?

Does the sum of the squares of the shorter sides equal the square of the large side?

$$3^2 + 6^2 = 9 + 36 = 45 = 9 \cdot 5 = (3\sqrt{5})^2 \quad \checkmark$$

Problem #18 Show that the equation $3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$ represents a sphere, and find its center and radius.

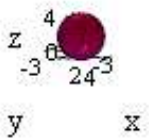
$$3x^2 + 3y^2 - 6y + 3z^2 - 12z = 10$$

$$x^2 + y^2 - 2y + z^2 - 4z = \frac{10}{3}$$

$$x^2 + (y^2 - 2y + 1) + (z^2 - 4z + 2) = \frac{10}{3} + 1 + 2$$

$$x^2 + (y - 1)^2 + (z - 2)^2 = \frac{19}{3}$$

Center is $(0, 1, 2)$, and radius is $\sqrt{\frac{19}{3}}$.



$$3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$$

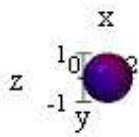
Problem #34 Describe in words the region of $\mathbb{R}^3$ represented by the inequality $x^2 + y^2 + z^2 > 2x$.

$$x^2 - 2x + y^2 + z^2 > 0$$

$$(x^2 - 2x + 1) + y^2 + z^2 > 1$$

$$(x - 1)^2 + y^2 + z^2 > 1$$

Therefore, $x^2 + y^2 + z^2 > 2x$ represents the region of $\mathbb{R}^3$ that is exterior to the sphere centered at $(1,0,0)$ with a radius 1.



$$(x - 1)^2 + y^2 + z^2 = 1$$

Problem #38 Write inequalities to describe a solid upper hemisphere of a sphere of radius 2 centered at the origin.

$$x^2 + y^2 + z^2 \leq 4, \text{ and } z \geq 0.$$