

Calculus II

Textbook: *Calculus Early Transcendentals*, Stewart, (8th edition).

Know how to decide between using:
IBP, Trig Identities, Trig Substitution, and Partial Fractions.

7.5 - Strategy for Integration

Putting it all together! In this section, you have to figure out which of the techniques of the previous sections to use for any particular integration problem. You also have to recall the various integration formulas. So, recall those from the review of section 7.1, in addition to the following...

$$\begin{aligned}\int \sec x \, dx &= \ln|\sec x + \tan x| + C & \int \csc x \, dx &= \ln|\csc x - \cot x| + C \\ \int \frac{dx}{x^2 - a^2} &= \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C & \int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C\end{aligned}$$

General Strategies:

- ♦ Simplify the integrand if possible
- ♦ Look for an obvious substitution
- ♦ Classify the integrand according to its form
 - a) Trigonometric functions
 - b) Rational functions
 - c) Integration by parts
 - d) Radicals
 - i) $\sqrt{\pm x^2 \pm a^2}$, trigonometric substitution
 - ii) $\sqrt[n]{ax + b}$, rationalizing substitution $u = \sqrt[n]{ax + b}$
- ♦ Try again
 - a) Try seemingly unlikely substitutions
 - b) Sometimes integration by parts can be used on single functions:
 $\int f(x) \, dx = \int 1 \cdot f(x) \, dx$. So let $u = f(x)$ and $dv = 1dx$!!
 - c) Manipulate the integrand, and perhaps something will present itself.
(e.g., partial fractions decomposition, among others).
 - d) Think of previous problems which had a similar form
 - e) Combine several of the methods above in succession.

Problem #4 Evaluate $\int \frac{\sin^3 x}{\cos x} \, dx$.

$$= \int \frac{\sin^2 x \sin x}{\cos x} = \int \frac{(1 - \cos^2 x) \sin x}{\cos x} dx$$

$$= \int \frac{1-u^2}{u} (-du) \text{ (with } u = \cos x, du = -\sin x dx \text{)}$$

$$= \int (u - \frac{1}{u}) du = \frac{1}{2} u^2 - \ln|u| + C = \frac{1}{2} \cos^2 x - \ln|\cos x| + C.$$

Problem #48 Evaluate $\int_0^1 x \sqrt{2 - \sqrt{1 - x^2}} dx$

Let $u = \sqrt{1 - x^2}$, so $u^2 = 1 - x^2$, and $2udu = -2xdx$.

Then, $\int_{x=0}^{x=1} x \sqrt{2 - \sqrt{1 - x^2}} dx = \int_{u=1}^{u=0} \sqrt{2 - u} (-udu)$.

Now let $v = \sqrt{2 - u}$, so $v^2 = 2 - u$, and $2v dv = -du$.

Thus, $\int_{u=1}^{u=0} \sqrt{2 - u} (-udu) = \int_{v=1}^{v=\sqrt{2}} v(2 - v^2)(2v dv) = \int_1^{\sqrt{2}} (4v^2 - 2v^4) dv = \left[\frac{4}{3} v^3 - \frac{2}{5} v^5 \right]_1^{\sqrt{2}}$

$$= \left(\frac{8}{3} \sqrt{2} - \frac{8}{5} \sqrt{2} \right) - \left(\frac{4}{3} - \frac{2}{5} \right) = \frac{16}{15} \sqrt{2} - \frac{14}{15} \approx 0.575.$$

Problem #64 Evaluate $\int \frac{1}{\sqrt{\sqrt{x}+1}} dx$.

Let $u = \sqrt{x} + 1$, so that $x = (u - 1)^2$ and $dx = 2(u - 1)du$.

Then $\int \frac{1}{\sqrt{\sqrt{x}+1}} dx = \int \frac{2(u-1)du}{\sqrt{u}} = \int (2u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}) du = \int \left(\frac{4}{3} u^{\frac{3}{2}} - 4u^{\frac{1}{2}} \right) du$

$$= \frac{4}{3} (\sqrt{x} + 1)^{\frac{3}{2}} - 4\sqrt{\sqrt{x} + 1} + C.$$
