

Exercises

§7.1-7.4: Multiclass Classification

Python: There is an optional starter script that supports Python-based problem(s) on this sheet. Use it as scaffolding: read the comments, complete the lines marked **TODO**, and compare the output with the mathematics requested in the problem(s).

Problem 1 A four-class model produces scores $(1.2, -0.4, 2.7, 2.1)$. Find the predicted class. If the true class is 4, compute the multiclass margin. Is the observation correctly classified?

Problem 2 Consider $f_1 = 1 + x_1$, $f_2 = 2 - x_1 + x_2$, $f_3 = -x_2$. Find the three pairwise boundaries: $f_1 = f_2$, $f_1 = f_3$, $f_2 = f_3$.

Problem 3 For the three-class example from class, $f_1 = 2 - x_1 - x_2$, $f_2 = x_1$, $f_3 = x_2$, classify $(1/2, 1/2)$, $(2, 1/2)$, $(1/2, 2)$. Compute the score vector at each point.

Problem 4 Suppose the true class is $y = 3$ and the score vector is $(1.8, 1.2, 1.5, 0.7)$. Compute: (a) the strongest competing class, (b) the multiclass margin, (c) the multiclass perceptron loss.

Problem 5 For one observation, $\tilde{\mathbf{x}} = (1, 2, -1)^T$, the true class is $y = 2$, but class 4 has the largest score. Using $\alpha = 0.1$, write the perceptron updates to $\mathbf{w}_2$ and $\mathbf{w}_4$. What happens to the other class parameter vectors?

Problem 6 Prove that adding the same number c to every class score does not change (a) the predicted class, (b) the multiclass margin.

Problem 7 Use Python and np to classify the rows of

```
scores = np.array([ [2.0, 1.0, 0.0],  
                    [0.2, 1.5, 1.1],  
                    [0.5, 0.5, 2.0],  
                    [3.0, 3.0, 1.0] ])
```

using class labels 1,2,3. What happens on the last row?

Problem 8 Suppose three class scores are $f_1 = f_2 = f_3 = 0$. (a) Apply a sigmoid independently to each score. (b) Add the resulting three numbers. (c) Explain why these cannot be probabilities for three mutually exclusive classes. What property must the next lesson's transformation enforce?