

Exercises

§5.3: Alternatives to Least Squares

Python: There is an optional starter script that supports Python-based problem(s) on this sheet. Use it as scaffolding: read the comments, complete the lines marked **TODO**, and compare the output with the mathematics requested in the problem(s).

Problem 1 For residual magnitudes 2, 5, 10, compute the absolute-error and squared-error penalties. If a residual doubles from 5 to 10, by what factor does each penalty increase?

Problem 2 Consider the constant model $\hat{y} = w$ for the data 2, 3, 4, 5, 30. Find the least-squares value of w . Find an LAD value of w .

Problem 3 For a general constant model, $g_2(w) = \frac{1}{P} \sum_{p=1}^P (w - y_p)^2$. Differentiate and show that the least-squares minimizer is $w = \bar{y}$.

Problem 4 For the data 1, 2, 2, 3, 20, compute the MAE for $w = 2$ and $w = 5.6$. Which value is preferred by LAD?

Problem 5 For the outlier regression example $x = (0, 1, 2, 3, 4)$, $y = (0, 1, 2, 3, 20)$, compare the two fitted lines $\hat{y} = -3.2 + 4.2x$ and $\hat{y} = x$. Compute the MAE of each. Which line does LAD prefer?

Problem 6 Why can ordinary Newton's method not be applied to LAD regression in exactly the same way that it was applied to least squares? What earlier optimization method could still be used without differentiability?

Problem 7 Modify the Python grid search from class by replacing the outlier value 20 with 15. Use the same parameter grids. What LAD parameter vector does the search return? Interpret the result.