

Exercises

§4.1,4.2: Second-Order Approximation

Python: There is an optional starter script that supports Python-based problem(s) on this sheet. Use it as scaffolding: read the comments, complete the lines marked **TODO**, and compare the output with the mathematics requested in the problem(s).

Problem 1 Use the second-order Taylor approximation for $g(w) = e^w$ about $w_0 = 0$ to estimate $e^{0.3}$. Compare with the first-order estimate and the actual value $e^{0.3} \approx 1.34986$.

Problem 2 Let $g(w_1, w_2) = w_1^2 + 4w_1w_2 + 5w_2^2$. Find ∇g and the Hessian H .

Problem 3 For $g(w_1, w_2) = w_1^2 + 3w_2^2$, use the second-order approximation at $(1, 1)$ to estimate $g(1.1, 0.9)$. Compare with the exact value.

Problem 4 Classify the stationary point at the origin for each function. (a) $g(w_1, w_2) = 2w_1^2 + 5w_2^2$, (b) $g(w_1, w_2) = -w_1^2 - 3w_2^2$, (c) $g(w_1, w_2) = w_1^2 - 4w_2^2$. Use the Hessian eigenvalues.

Problem 5 Use Python to find the eigenvalues of $H = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$. Is H positive definite, negative definite, or indefinite?

Problem 6 A quadratic cost has Hessian eigenvalues $\lambda_{\min} = 2, \lambda_{\max} = 200$. Find its condition number. Explain what this suggests about gradient descent compared with a quadratic whose Hessian eigenvalues are 2 and 4.