

Introduction to Statistics I

Textbook: Elementary Statistics (4th Edition, by Navidi and Monk), and Workshop Statistics (4th Edition by Rossman and Chance).

Previous Lecture

- ◆ Definitions in Probability
- ◆ Law of Large Numbers
- ◆ Tree Diagrams
- ◆ Empirical Method



§5.2: Probability. Addition Rule and Rule of Compliments

Sometimes you are interested in an outcome that depends on two (or more) different events.

For instance, let's say you are on the basketball team, and this will mean you'll miss about one fifth of your Friday classes, but you're not sure which Fridays yet.

Also, you know that about 25% of the Fridays, your math class will have a pop quiz.

You are interested in the probability that you will miss a quiz during the semester.



Notice the outcome depends upon two events: when you'll miss class, and when the quizzes occur.

Definition: A *compound event* is an event that is formed by combining two or more events.

This could be the probability that several events all happen, or that some events happen while others don't, or that none of the events happen.

Contingency Tables

Contingency tables are used to examine relationship between **two categorical vars**.

When gathering data, we classify obs-units **two ways**.

Example RQ: Are women more likely to report feeling rushed than men? Explanatory/Response?

Explanatory Var: male or female.

Response Var: how frequently they feel rushed.



Let's assume we randomly ask people in Syracuse to tell us their gender and how rushed they feel.

	Males	Females	Total
Always	116	188	304
Sometimes	239	284	513
Never	82	78	160
Total	427	550	977

Explanatory categories listed across **top/columns**.

Response categories listed down **side/rows**.

in each cell is # of obs-units who fit into both row and column categories.

What’s the probability that a randomly selected person is female and never feels rushed?

$$\frac{\text{\# of people of interest}}{\text{Total \# in population}} = \frac{188}{977} \approx 0.1924 \text{ or } 19.2 \%$$

What is the probability that randomly selected person sometimes feels rushed?

$$\frac{\text{\# of people of interest}}{\text{Total \# in population}} = \frac{239+284}{977} = \frac{513}{977} \approx 0.525 \text{ or } 52.5\%$$

The General Addition Rule

Below, we will look at the probability that **at least one event** occurs.

Notation: $P(A \text{ or } B) = P(A \text{ occurs or } B \text{ occurs or both occur})$.

What if we wanted to know the probability that someone in Syracuse was either female, or always felt rushed?

How can we calculate this?

	Males	Females	Total
Always	116	188	304
Sometimes	239	284	513
Never	82	78	160
Total	427	550	977

In the table, you’ll notice I bolded both the females and those who always felt rushed.

Looking at the totals, we have 304 people who always feel rushed, and 550 females.

Naïvely, we might add the probabilities of these things being true:

$$P(\text{Always}) + P(\text{Female}) = \frac{\text{\#Always}}{\text{Total}} + \frac{\text{\#Female}}{\text{Total}} = \frac{304}{977} + \frac{550}{977} \approx 0.874. \text{ (INCORRECT)}$$

What error have we made?

Yes, we counted the **females** who feel **always rushed** twice!

So, we must remember to subtract that probability from our calculation.

$$\begin{aligned} P(\text{Always OR Female}) &= P(\text{Always}) + P(\text{Female}) - P(\text{Always and Female}) \\ &= \frac{\text{\#Always}}{\text{Total}} + \frac{\text{\#Female}}{\text{Total}} - \frac{\text{\#Always AND \#Female}}{\text{Total}} = \frac{304}{977} + \frac{550}{977} - \frac{188}{977} \approx 0.682. \end{aligned}$$

General Addition Rule: For any two events A and B , $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Mutually Exclusive Events

A study of a busy intersection shows that, over many random observations the light is red 40 percent of the time, green 45 percent of the time, and yellow 15 percent of the time. What is the probability that at any particular time the light is red or green?

If we denote the event that the light is red A , and denote the event that the light is green B , then recall that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, but in this case observe that $P(A \text{ and } B) = 0$.

Definition: Two events are said to be *mutually exclusive* if it is impossible for both events to occur.

The Addition Rule for Mutually Exclusive Events:

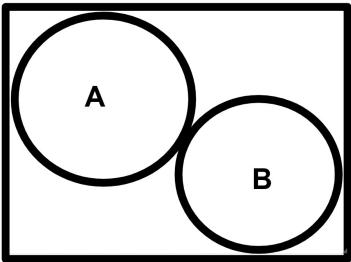
If A and B are mutually exclusive events, then: $P(A \text{ or } B) = P(A) + P(B)$.

In general, three or more events are mutually exclusive if only one of them can happen.

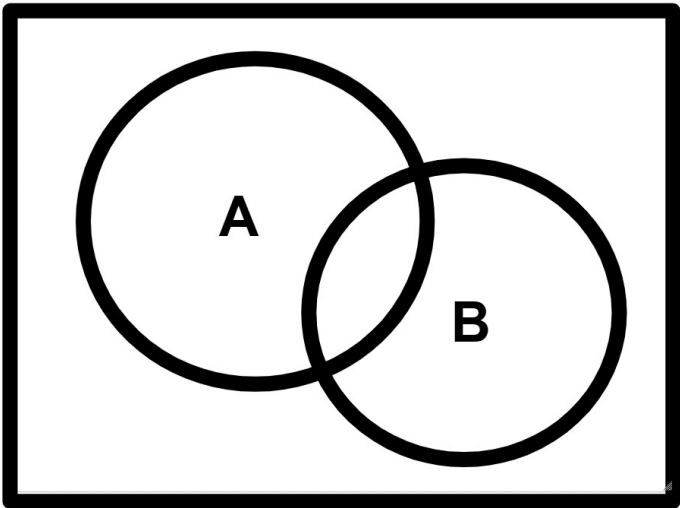
If $A, B, C, \dots$ are mutually exclusive, then: $P(A \text{ or } B \text{ or } C \text{ or } \dots) = P(A) + P(B) + P(C) + \dots$.

Back to example: $P(A \text{ or } B) = P(A) + P(B) = 0.45 + 0.4 = 0.95$.

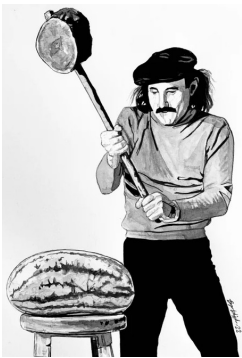
Venn diagrams are often used to depict sample spaces. For instance, in the situation we would draw:



But what if the light was broken, and 10 percent of the time it showed both red and green? This would no doubt cause some accidents, but what would our Venn diagram look like?



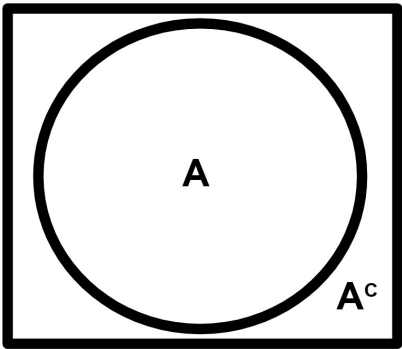
Complements



You have front row tickets to see your favorite comedian, but one of his skits that you don't enjoy is when he uses a large hammer to crush a watermelon, sending watermelon juice into the audience. So, you wonder what the probability is that he will skip that particular skit. You have read online that this comedian does not do this skit approximately 28 percent of the time. What are the chances you'll leave the auditorium covered in watermelon juice?

Definition: If A is any event, the *complement of A* is the event that A does not occur. The complement of A is denoted A^c .

The Rule of Complements: $P(A^c) = 1 - P(A)$.



Back to Our Example: To calculate our probability, let A be the event that the comedian does not do the watermelon skit. Therefore, the probability that you will be covered in watermelon juice is:

$$\begin{aligned} P(A^c) &= 1 - P(A) \\ &= 1 - 0.28 = 0.72. \end{aligned}$$

What did we learn?



- ◆ Contingency Tables
- ◆ General Addition Rule
- ◆ Mutually Exclusive and Complementary Events