

Introduction to Statistics I

Textbook: Elementary Statistics (4th Edition, by Navidi and Monk), and Workshop Statistics (4th Edition by Rossman and Chance).

§5.1: Basic Concepts of Probability

Life is complicated, and even the simplest of processes are unpredictable.



For instance, no one can predict what number will come up when I roll a 6 sided die.

However, we can be sure that if I rolled the die many many times, on average, the number "1" will come up $\frac{1}{6}$ of the time. In other words, there's a $\frac{1}{6}$ th probability that the die will land on "1" for any particular roll.

Definition: A *probability experiment* is one in which we do not know what any individual outcome will be, but we do know how a long series of repetitions will come out.

Definition: The *probability* of an event is the proportion of times the event occurs in the long run, if a probability experiment is repeated over and over again.



For instance, I could roll the die 1000 times, and 160 of those times, it might land on "1".

Dividing, we can calculate the proportion of times it landed on "1": $\frac{160}{1000} = 0.160$.

We can see that this is very close to $\frac{1}{6} \approx 0.167$.

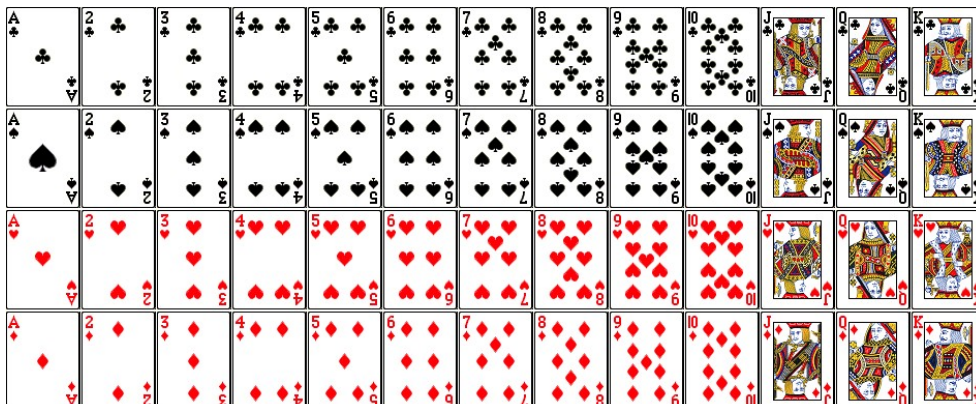
This phenomenon is encapsulated in the following law:

The Law of Large Numbers says that as a probability experiment is repeated again and again, the proportion of times that a given event occurs will approach its probability.



To fill up our probability toolbox, we need a few more definitions.

Definition: A *sample space* contains all the possible outcomes of a probability experiment.



Sample Space for drawing a card from a standard deck of cards.

Definition: An event is an outcome or a collection of outcomes from a sample space.

Example: Drawing an ace is an event. So is drawing the ace of hearts. So is drawing a pair of kings.

Definition: A *probability model* for a probability experiment consists of a sample space, along with a probability for each event. If A denotes an event, the probability of A is denoted $P(A)$.

Example: If we define A as drawing the ace of hearts, then $P(A) = \frac{1}{52} \approx 0.0192$ or 1.9 %.

Computing Probabilities with Equal Likely Outcomes (e.g., fair coin or die):

If a sample space has n equally likely outcomes, and an event A has k outcomes, then:

$$P(A) = \frac{\text{\# of outcomes in } A}{\text{\# of outcomes in the sample space}} = \frac{k}{n}.$$

Example: Observe that the probability of drawing any particular card (e.g., queen of hearts) is the same: $\frac{1}{52}$.

So when drawing one card, what is the probability of drawing a jack? Let’s denote this event A .

$$P(A) = \frac{\text{\# of outcomes in } A}{\text{\# of outcomes in the sample space}} = \frac{4}{52} \approx 0.077 \text{ or } 7.7\%.$$

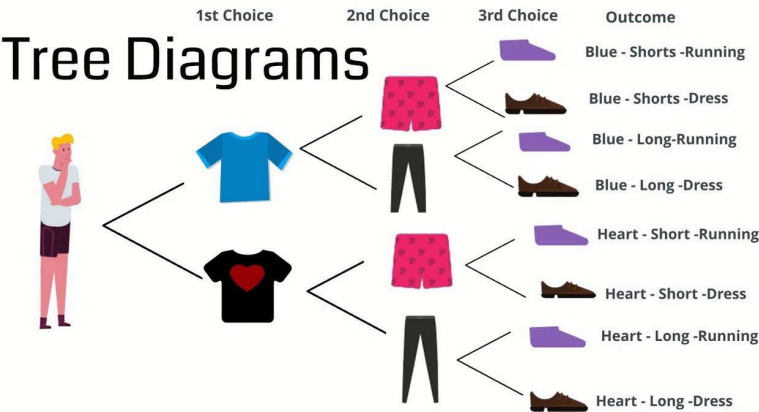
Note: The probability of an event is always between 0 and 1.

In other words, for any event A , we have $0 \leq P(A) \leq 1$.

- ♦ If A cannot occur, then $P(A) = 0$.
- ♦ If A is certain to occur, then $P(A) = 1$.

Example: Each day Philip wakes up, he randomly chooses what to wear. He owns two tops: one blue, and one with a heart logo. He owns two bottoms: shorts or pants. And he owns two types of shoes: running and dress. What is the probability he will wear his heart shirt with dress shoes?

We can use *tree diagrams* to visualize these type of probability models:



Observe that there are a total of 8 outcomes.

Also observe that 2 of them involve his wearing his heart shirt and his dress shoes.
And since he chooses his outfit randomly, each outcome is as likely as the other.
Therefore, the probability of him wearing his heart shirt and his dress shoes is $\frac{2}{8} = \frac{1}{4}$ or $P(A) = 0.25 = 25\%$.

❗ Sampling from a population is a probability experiment.

The population is the sample space, and the members of the population are equally likely outcomes.

For this reason, the **ideas of probability are fundamental to statistics**.

Approximating Probabilities with the Empirical Method

The Empirical method consists of repeating an experiment a large number of times and using the proportion of times an outcome occurs to approximate the probability of the outcome.



Example: A nationwide survey of 1001 registered voters was conducted in 2022. They were asked if they supported or opposed in national paid family and medical leave program. 801 respondents said they supported such a program, while 120 were opposed (80 were unsure). Approximate the probability that a registered voter would support such a program.

The sample proportion was: $\frac{801}{1001} \approx 0.80$ or 80%.

And so, our best approximation of the larger population is that the probability that a registered voter would support such a program is 80 percent.



Instead of doing experiments by hand, one can sometimes use computers to simulate the experiment, and thereby manage to increase the number of experiments performed quite significantly.
(e.g., if you roll 9 six-sided dice, how likely are you to roll a 42? Have a computer simulate it 10,000 times!)

Activity:

What did we learn?

- ◆ Definitions in Probability
- ◆ Law of Large Numbers
- ◆ Tree Diagrams
- ◆ Empirical Method



