

Worksheet 10 — Eigenvalues, eigenvectors, and diagonalization

Sections 8.2–8.3 • Complex eigenvalues included • Student worksheet

Learning goals. Use eigenvalue identities, diagonalize when a full eigenbasis exists, and work comfortably with conjugate complex eigenpairs of a real matrix.

Name(s): _____

Section/group: _____

Core practice

1. Eigenvalue identities

Let $Av = \lambda v$ with $v \neq 0$. Prove: (a) λ^2 is an eigenvalue of A^2 ; (b) if A is invertible, then $\lambda \neq 0$ and λ^{-1} is an eigenvalue of A^{-1} . Identify the eigenvector in both parts.

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2. A triangular matrix with a full eigenbasis

Find all eigenvalues and a basis for each eigenspace. Use the results to diagonalize A . Check the sum and product of the eigenvalues against $\text{trace}(A)$ and $\det(A)$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

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3. A real matrix with complex eigenvalues

Find the eigenvalues and complex eigenvectors of A . Diagonalize A over $\mathbb{C}$, and explain why no real diagonalization is possible.

$$A = \begin{bmatrix} 2 & -5 \\ 1 & 2 \end{bmatrix}.$$

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4. Algebraic versus geometric multiplicity

For the matrix below, find the eigenspaces for $\lambda = 1$ and $\lambda = 2$, compare algebraic and geometric multiplicities, and decide whether the matrix is diagonalizable.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

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