

Worksheet 9 — Linear maps, geometry, and change of basis

Sections 7.1–7.2 • Student worksheet

Learning goals. Test linearity, read a planar transformation geometrically, and compute a matrix representation in a basis chosen to reveal the map's structure.

Name(s): _____

Section/group: _____

Core practice

1. Three maps: which are linear?

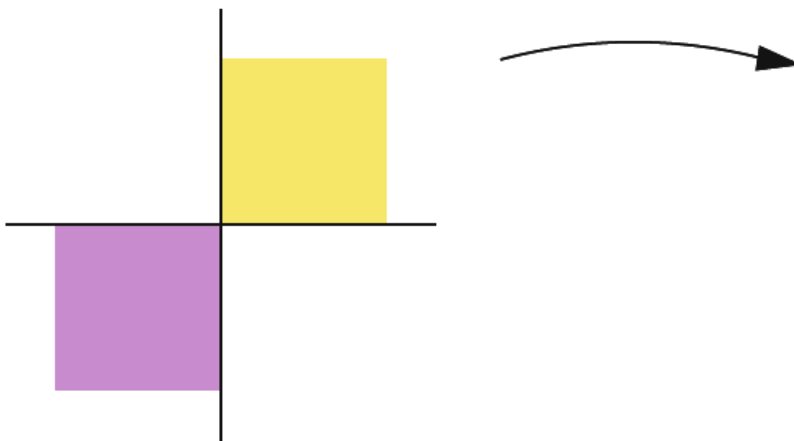
For each map, decide whether it is linear. If it is linear, verify additivity and homogeneity or identify its matrix. If it is not, give one decisive failure.

- (a) $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $L(x, y) = (x+2, x+y)$.
- (b) $D: P_3 \rightarrow P_3$, $D(f) = 2f + f'$.
- (c) $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, $F(x, y) = (3x, 2y, xy)$.

Workspace

2. Track two squares through a shear

The picture shows the unit square in the first quadrant and a congruent square in the third quadrant. Consider $T(x, y) = (x - y/2, y)$.



(a) Find the standard matrix of T . (b) Map all four vertices of each square. (c) Sketch the two image parallelograms and describe what the map preserves.

Workspace

3. Choose a basis that diagonalizes the action

The standard matrix of $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is A . Let $\beta = (v_1, v_2)$ with $v_1 = (2, 1)^T$ and $v_2 = (-1, 1)^T$. Find $[L]_{\beta \leftarrow \beta}$ two ways: by computing $S^{-1}AS$, and by observing the action on the basis vectors.

$$A = \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix}.$$

Workspace

Extension / challenge

4. Extension: composing two reflections

Let S be reflection across the line $y = x$, and let T be reflection through the origin. Find the standard matrices, determine whether the maps commute, and identify the geometry of $S \circ T$.

Workspace

