

Worksheet 7 — Orthogonality and Gram–Schmidt

Sections 4.1–4.2 • Student worksheet

Learning goals. Use orthonormal coordinates, execute Gram–Schmidt exactly, detect dependence through a zero residual, and work with a matrix-defined inner product.

Name(s): _____

Section/group: _____

Core practice

1. Coordinates in an orthonormal basis

Suppose $\{v_1, v_2, v_3\}$ is an orthonormal basis of a three-dimensional inner-product space and a vector w satisfies $\langle w, v_1 \rangle = 1$, $\langle w, v_2 \rangle = 1/3$, and $\langle w, v_3 \rangle = 7/9$. Express w in this basis and find $\|w\|$.

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2. Gram–Schmidt in $\mathbb{R}^3$

Apply Gram–Schmidt in the listed order to produce an orthonormal basis for the span of

$$w_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Keep exact radicals and show every projection coefficient.

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3. Dependence revealed by a zero residual

Run Gram–Schmidt on

$$w_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}.$$

What happens at the third step? Explain what the result says about the original list and identify a basis for its span.

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Extension / challenge

4. Extension: Gram–Schmidt in a K -inner product

Let $\langle x, y \rangle_K = x^T K y$, where

$$K = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 3 & -3 \\ 0 & -3 & 11 \end{bmatrix}.$$

Verify that K is positive definite. Apply Gram–Schmidt to e_1, e_2, e_3 using this inner product. It is enough to give a K -orthogonal basis and its squared K -norms.

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