

Worksheet 6 — Quadratic forms and unconstrained optimization

Sections 3.3–3.5 • Student worksheet

Learning goals. Move between matrices and quadratic expressions, use LDL^T to classify a form, and locate the unique minimizer of a positive-definite quadratic.

Name(s): _____

Section/group: _____

Core practice

1. Read a quadratic form from a matrix

Let $A = (a_{ij})$ be a real 3×3 matrix and $x = (x_1, x_2, x_3)^T$. Expand $x^T A x$. Which entries of A control the coefficient of $x_i x_j$ when $i \neq j$? Explain why every quadratic form may be represented by a symmetric matrix.

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2. Classify a form using LDL^T

Factor the symmetric matrix K as $K = LDL^T$, where L has diagonal entries 1. Use the diagonal factor to decide whether K is positive definite, negative definite, or indefinite.

$$K = \begin{bmatrix} 5 & 10 & -5 \\ 10 & 19 & -13 \\ -5 & -13 & 3 \end{bmatrix}.$$

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3. A closure property for positive-definite matrices

Suppose K and H are symmetric positive-definite matrices of the same size. Prove that $K + H$ is positive definite.

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4. Complete the square in matrix form

Find the global minimizer and minimum value of

$$p(x, y, z) = x^2 + y^2 + yz + z^2 + x + y - z.$$

Write the expression as $u^T Ku - 2f^T u + c$, verify that K is positive definite, and solve $Ku^* = f$.

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