

Worksheet 4 — Subspaces, span, independence, and bases

Sections 2.2–2.4 • Student worksheet

Learning goals. Apply the subspace test, translate polynomials into coordinate vectors, and describe spans using equations and parameters.

Name(s): _____

Section/group: _____

Core practice

1. A subspace defined by trace

Let W be the set of all 3×3 real matrices with trace zero. Prove that W is a subspace of the vector space of 3×3 matrices.

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2. A polynomial basis test

In P_2 , consider $f_1(x) = x^2 - 3$, $f_2(x) = 2 - x$, and $f_3(x) = (x - 2)^2$. Decide whether these polynomials form a basis of P_2 . Show your coordinate-matrix calculation relative to $(1, x, x^2)$.

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3. Convert a span description into an equation

Let $W = \text{span}\{(-1, 3, 2), (0, 1, 1)\}$. Find a single equation in a , b , and c that characterizes when (a, b, c) belongs to W . When the condition holds, express the coefficients of the spanning vectors in terms of a and b .

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Extension / challenge

4. Extension: rapid subspace triage

For each set, decide whether it is a subspace of the ambient vector space. Identify the first subspace condition that succeeds or fails.

- (a) $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$.
- (b) $\{(x,y,z) \in \mathbb{R}^3 : x - 2y + z = 0\}$.
- (c) $\{f \in C[-1,1] : \int_{-1}^1 f(x) \, dx = 0\}$.
- (d) $\{0\}$ together with all polynomials of odd degree.

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