

Worksheet 3 — Determinants and vector-space reasoning

Sections 1.9 and 2.1 • Student worksheet

Learning goals. Use determinant identities in proofs and verify vector-space structure in an unfamiliar setting.

Name(s): _____

Section/group: _____

Core practice

1. An involution has a restricted determinant

Suppose A is a real square matrix satisfying $A^2 = I$. Prove that A is invertible and that $\det(A)$ must equal 1 or -1 .

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2. Odd-dimensional skew-symmetric matrices

A real matrix M is skew-symmetric when $M^T = -M$. Prove that every skew-symmetric matrix of odd size is singular.

Your proof should explicitly use both $\det(M^T) = \det(M)$ and the effect of multiplying an $n \times n$ matrix by -1 .

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3. A vector space of infinite sequences

Let $\mathbb{R}^\infty$ be the set of all infinite real sequences $x = (x_1, x_2, \dots)$. Define addition and scalar multiplication componentwise.

- (a) State the zero vector and the additive inverse of x . (b) Verify four vector-space axioms of your choice. (c) Explain why the remaining axioms follow from the corresponding properties of real numbers.

Extension / challenge

4. Extension: why the exponent is n

Give two explanations of $\det(-M) = (-1)^n \det(M)$: one using multilinearity in the rows and one using $\det((-I)M)$.

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